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SUSPENDED SUBSTRATE RESONATOR DESIGN

by

Sudjiwo

December 1990

Thesis Advisor Harry A. Atwater

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The open-end resonator in suspended substrate transmission line was analyzed in terms of two open-end discontinuities on a line segment. A procedure for computing the resonant frequency and constant of propagation was demonstrated using a full wave analysis. The fringing capacitance was computed using an equivalent length extension model and a transmission line circuit model. The characteristic equation was derived using Galerkin's method applied in the Fourier transform domain. The calculation has been carried out in two increasing orders of approximation, and the results compared. The discontinuity capacitance at the resonator open ends was calculated for a range of line dimensions and substrate dielectric constants. The dispersive propagation constant of suspended substrate line was also calculated by the Galerkin method.

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SUSPENDED SUBSTRATE RESONATOR DESIGN

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ABSTRACT

The open-end resonator in suspended substrate transmission line was analyzed in terms of two open-end discontinuities on a line segment. A procedure for computing the resonant frequency and constant of propagation was demonstrated using a full wave analysis. The fringing capacitance was computed using an equivalent length extension model and a transmission line circuit model. The characteristic equation was derived using Galerkin's method applied in the Fourier transform domain. The calculation has been carried out in two increasing orders of approximation, and the results compared. The discontinuity capacitance at the resonator open ends was calculated for a range of line dimensions and substrate dielectric constants. The dispersive propagation constant of suspended substrate line was also calculated by the Galerkin method.

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I. INTRODUCTION

The analysis of planar transmission lines in the Fourier transform domain or spectral domain is superior to many numerical methods in the spatial domain. The analysis in the Fourier transform domain was used earlier by Yamashita and Mittra [Ref. 4] for computation of the characteristic impedance and phase velocity of a microstrip line based on a quasi-TEM approximation. A variational method was used in the Fourier transform domain to calculate the line capacitance from the assumed charge density. This is a low-frequency approximation neglecting longitudinal electric and magnetic fields supported by the strip transmission line having two dielectric media in its cross section, or inhomogeneous line.

As the operating frequency is increased, dispersion characteristics of the inhomogeneous line become important for precise designs. This requirement has led to the full wave analysis of microstrip lines, represented by the work of Denlinger [Ref. 15], who solved the integral equations using a Fourier transform technique. The solution by his method, however, strongly depends on the assumed current distributions on the strip in process of solution. To avoid this difficulty and permit systematic improvement of the solution for the current components to a desired degree of accuracy, a new method was presented by Itoh and Mittra and commonly called the Spectral Domain Approach (SDA). In SDA, Galerkin's method is used to yield a homogeneous system of equations to determine the propagation constant and the characteristic frequencies from which the equivalent circuit is derived.

In each of these methods the Fourier transform is taken along the coordinate axes in the plane of the strip. By virtue of the Fourier transform domain analysis and Galerkin's method, SDA has several features:

- Easy formulation in the form of a pair of algebraic equations.
- Variational nature in determination of the propagation constant.

The Spectral Domain Approach is applicable to the following structures:

- Most planar transmission lines such as microstrip, finline, and coplanar waveguide in multilayer configurations.
- Both open and enclosed structures.
- Slow-wave lines with lossy dielectric materials.
- Resonators of planar configurations.

An efficient formulation of the SDA is achieved in the present work by enclosing the suspended substrate in a metallic shield enclosure. In this way the Fourier transform becomes the finite Fourier transform, calculated as a summation. Singularities of the integrand can be avoided, and the truncated summations have acceptable convergence behavior. As applied here, the analysis assumes loss-free conductors and dielectric media, and also assumes infinitesimal thickness for the strip conductors. The resonant frequencies are obtained by numerically solving the characteristic equation, and equivalent open-circuit capacitances are obtained using the end-effect and the transmission-line models. The details of the analysis method will appear in Chapter II of this thesis.

In Chapter III, discussions are presented concerning the accuracy of the solution for the resonant frequencies, the propagation constants, and the end-effect at the open end of the microstrip structure by calculation of the fringing capacitance. These solutions for various orders of ap-

proximation are compared, based on the assumptions for various current distributions in x and z - directions.

II. SPECTRAL DOMAIN APPROACH FOR SHIELDED SUSPENDED SUBSTRATE RESONATOR

A. INTRODUCTION

The shielded suspended substrate resonator to be analyzed is shown in Figure 1. A rectangular strip of width w and length l is placed on the suspended substrate which is separated from the ground plane by an air gap. The sides and the top of the structure are enclosed with metallic shielding walls. Thus the entire structure is considered to be the suspended microstrip resonator located in enclosed partially filled waveguide. It is assumed that the thickness of the strip is negligible and that all the media and conductors are lossless. The shielding waveguide has dimensions 3.2 mm X 1.575 mm X 100.0 mm, which corresponds to a system in actual practice. For simplicity, the strip is assumed to be symmetrically located.

The operating frequency is chosen to be below cutoff of the shielding waveguide partially filled with substrate material, to avoid interaction with waveguide-mode resonances. From the formula in [Ref. 10] the cutoff frequency for the structure in Figure 1 is shown in Table 1.

Table 1. THE CUTOFF FREQUENCY OF SUSPENDED SUBSTRATE LINE

Dielectric constant (ε_r)	Frequency cutoff (GHZ)
2.2 (duroid)	44.7574
9.8 (alumina = Al_2O_3)	43.3333

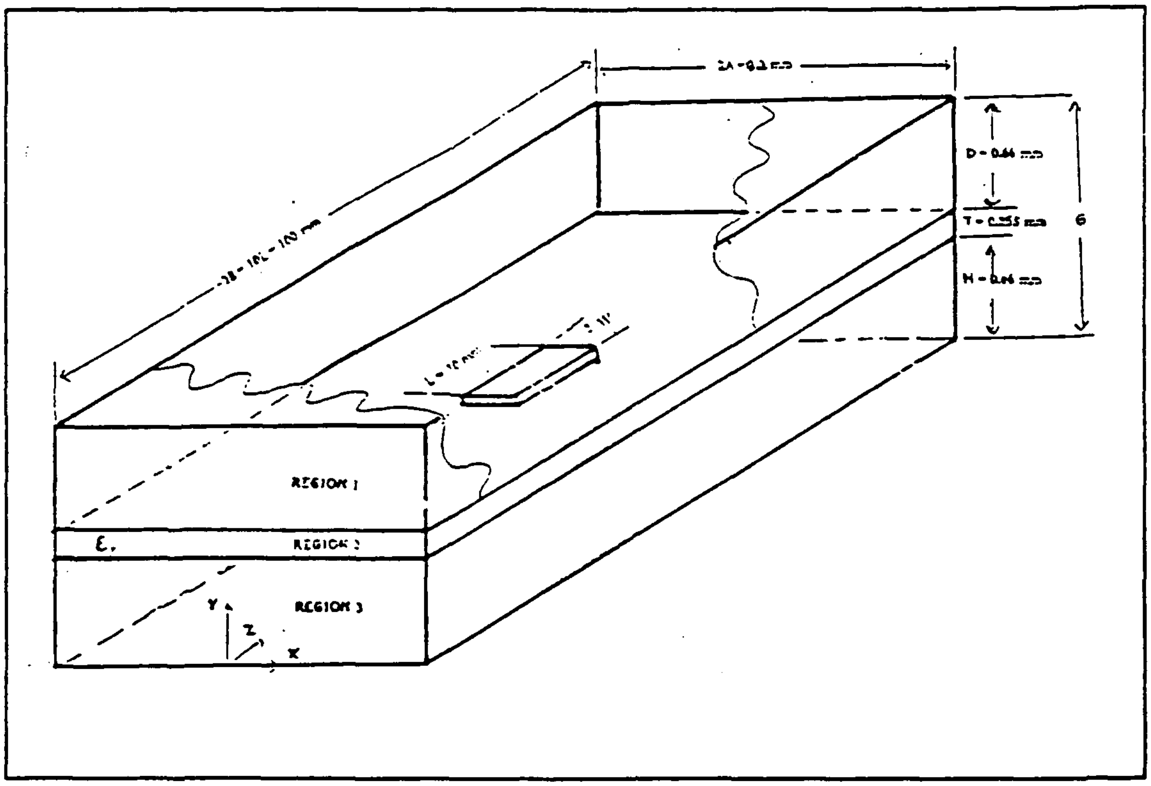

Figure 1. Shielded suspended substrate resonator

B. METHOD OF SOLUTION FOR RESONANT FREQUENCY

1. Field Equations

The analysis of field equations on strip transmission lines of the type considered here has been carried out by Uwano and Itoh [Ref. 2]. These authors treated the problem by applying Galerkin's method of moments in the Fourier transform domain. Their work is summarized in a set of Green's function equations as follows

$$\tilde{Z}_{zz}\tilde{J}_z + \tilde{Z}_{zx}\tilde{J}_x = \tilde{E}_z \tag{1a}$$

$$\tilde{Z}_{xz}\tilde{J}_z + \tilde{Z}_{xx}\tilde{J}_x = \tilde{E}_x \tag{1b}$$

Where the tilde over the symbols indicates the Fourier transformed quantities. Here, $\tilde{J}_z$ and $\tilde{J}_x$ are the transformed z and x components of the currents on the strip conductor. $\tilde{E}_z$ and $\tilde{E}_x$ are the components of electric field tangential to the substrate surface. $\tilde{Z}_{zz}$, $\tilde{Z}_{zx}$, $\tilde{Z}_{xz}$, $\tilde{Z}_{xx}$ are the Green's impedance functions, and are defined as the following :

$$\tilde{Z}_{zz} = -\frac{1}{\alpha^2+\beta^2}\left[\beta^2\tilde{Z}_e + \alpha^2\tilde{Z}_h\right] \tag{1c}$$

$$\tilde{Z}_{zx} = -\frac{\alpha\beta}{\alpha^2+\beta^2}\left[\tilde{Z}_e - \tilde{Z}_h\right] \tag{1d}$$

$$\tilde{Z}_{xz} = \tilde{Z}_{zx} \tag{1e}$$

$$\tilde{Z}_{xx} = -\frac{1}{\alpha^2+\beta^2}\left[\alpha^2\tilde{Z}_e + \beta^2\tilde{Z}_h\right] \tag{1f}$$

$$\tilde{Z}_e = \frac{\gamma_{y2}Ct_3 + \gamma_{y3}Ct_2}{Ct_2Ct_3 + Ct_1Ct_3\gamma_{y2}/\gamma_{y1} + Ct_1Ct_2\gamma_{y3}/\gamma_{y1} + \gamma_{y3}/\gamma_{y2}} \tag{1g}$$

$$\tilde{Z}_h = \frac{\gamma_{z2}Ct_2 + \gamma_{z3}Ct_3}{\gamma_{z1}\gamma_{z2}Ct_1Ct_2 + \gamma_{z1}\gamma_{z3}Ct_1Ct_3 + \gamma_{z2}\gamma_{z3}Ct_2Ct_3 + \gamma_{z2}^2} \tag{1h}$$

$$Ct_1 = coth\ \gamma_1 h \tag{1i}$$

$$Ct_2 = coth\ \gamma_2 t \tag{1j}$$

$$Ct_3 = coth\ \gamma_3 d \tag{1k}$$

$$\gamma_l^2 = \alpha^2 + \beta^2 - k_l^2 \tag{1l}$$

$$k_l^2 = \omega^2 \mu_l \varepsilon_l \tag{1m}$$

$$\gamma_{yl} = \frac{\gamma_l}{y_l} \tag{1n}$$

$$\gamma_{zl} = \frac{\gamma_l}{z_l} \tag{1o}$$

$$y_l = j\omega\varepsilon_l \tag{1p}$$

$$z_l = j\omega\mu_l \tag{1q}$$

$$\varepsilon_l = \varepsilon_r \varepsilon_o \tag{1r}$$

$$\mu_l = \mu_r \mu_o \tag{1s}$$

Where subscripts $l = 1,2,3$ refer to the corresponding regions 1,2,3, ω is the operating frequency, and ε_o and μ_o are the free-space permittivity and permeability, respectively. In this thesis, the Fourier transform is carried out in the bounded region interior to the enclosed resonator. In order to examine the layered structures in Figure 1 with added bounding vertical electric and magnetic walls at $x = \pm 1.6mm$, $z = \pm 50mm$, the finite Fourier transform should be used [Ref. 20]. There we have the two-dimensional finite Fourier transform pairs

$$\tilde{f}(\alpha_n, y, \beta_m) = \int_{-1.6}^{1.6} \int_{-50}^{50} f(x, y, z) e^{-j\alpha_n x} e^{-j\beta_m z} dx dz \tag{2a}$$

$$f(x,y,z) = \frac{1}{4ab} \sum_{\alpha_n} \sum_{\beta_m} \tilde{f}(\alpha_n, y, \beta_m) e^{j\alpha_n x} e^{j\beta_m z} d\alpha_n d\beta_m \tag{2b}$$

Where α_n and β_m are the discrete transform variables for the dominant mode and are defined by, $\alpha_n = (n - 1/2)\pi/a$ for E_z even, $-H_z$ odd(in x) modes, and $\beta_m = (m - 1/2)\pi/b$ for E_z even, $-H_z$ odd(in z) modes. Note that b is the half length of the resonator and is defined to be 5 times the length of strip, and a is the half width of the resonator. From (2b) we have

$$f(x,y,z) = \frac{\pi^2}{(2ab)^2} \sum_{\alpha_n} \sum_{\beta_m} \tilde{f}(\alpha_n, y, \beta_m) e^{j\alpha_n x} e^{j\beta_m z} \tag{2c}$$

Equations (1a,b) contain four unknowns $\tilde{J}_z$, $\tilde{J}_x$, $\tilde{E}_z$, and $\tilde{E}_x$. Two unknowns $\tilde{E}_z$ and $\tilde{E}_x$, however, can be eliminated by applying Galerkin's method in the spectral domain. The first step is to expand the unknowns $\tilde{J}_z$ and $\tilde{J}_x$ in terms of known basis functions $\tilde{J}_{zj}$ and $\tilde{J}_{xk}$

$$\tilde{J}_z(\alpha_n, \beta_m) = \sum_{j=1}^{N2} c_j \tilde{J}_{zj}(\alpha_n, \beta_m) \tag{3a}$$

$$\tilde{J}_x(\alpha_n, \beta_m) = \sum_{k=1}^{N1} d_k \tilde{J}_{xk}(\alpha_n, \beta_m) \tag{3b}$$

Where c_j and d_k are unknown coefficients. The basis functions must be chosen to approximate the true but unknown distributions on the strip.

Since the current is nonzero only on the strip, therefore each basis function must be chosen so that it is nonzero only on the strip. After substituting (3) into (1a,b) and taking inner products of the resulting equations with the basis functions $\tilde{J}_{zl}$ and $\tilde{J}_{xl}$ for different values of i, this process yields the equations,

$$\sum_{j=1}^{N1} K_{ij}^{(1,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(1,2)} d_k = 0 \quad , i = 1,2,3,..,N2. \tag{4a}$$

$$\sum_{j=1}^{N1} K_{ij}^{(2,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(2,2)} d_k = 0 \quad , i = 1,2,3,..,N1. \tag{4b}$$

Where from definition of inner products associated with the Fourier transform defined by (2), equations (4) lead to a set of homogeneous equations forming the solution matrix for the c_j and d_k . The matrix elements are

$$K_{ij}^{(1,1)} = c_j \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{zi}(\alpha_n,\beta_m)\tilde{Z}_{zz}(\alpha_n,\beta_m)\tilde{J}_{zj}(\alpha_n,\beta_m) \tag{5a}$$

$$K_{ik}^{(1,2)} = d_k \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{zi}(\alpha_n,\beta_m)\tilde{Z}_{zx}(\alpha_n,\beta_m)\tilde{J}_{xk}(\alpha_n,\beta_m) \tag{5b}$$

$$K_{ij}^{(2,1)} = c_j \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{xi}(\alpha_n,\beta_m)\tilde{Z}_{xz}(\alpha_n,\beta_m)\tilde{J}_{zj}(\alpha_n,\beta_m) \tag{5c}$$

$$K_{ik}^{(2,2)} = d_k \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \tilde{J}_{xi}(\alpha_n,\beta_m)\tilde{Z}_{xx}(\alpha_n,\beta_m)\tilde{J}_{xk}(\alpha_n,\beta_m) \tag{5d}$$

The right hand side of (1a,b) can be eliminated in Galerkin's process via the application of Parseval's relation, because the current $\tilde{J}_{zi}(x)$, $\tilde{J}_{xi}(x)$ and the field components $\tilde{E}_z(x,D + T)$, $\tilde{E}_x(x,D + T)$ are nonzero in the complementary regions of x. Note that the solution matrix is symmetrical. In order to have non-trivial values for c_k , d_j , the determinant of the matrix must be zero. From this we have the solution for resonant frequency. The accuracy of the solution can be systematically improved by increasing the number of basis function (N1 + N2) and by solving a larger size matrix equation. However, if the first few basis functions are chosen so as to approximate the actual unknown current distribution reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution, resulting in numerical efficiency. Moreover, the numbers of basis functions for $J_z(z)$ and $J_x(z)$ should be equal [Ref. 18]. Hence the choice of basis functions is important from the numerical point of view. In actual computations for the dominant mode [Ref. 1], $\tilde{J}_{zi}$ and $\tilde{J}_{xi}$ have been chosen to be product functions of the form:

$$\tilde{J}_{zi}(\alpha_n,\beta_m) = \tilde{J}_1(\alpha_n)\tilde{J}_2(\beta_m) \tag{6a}$$

$$\tilde{J}_{xi}(\alpha_n,\beta_m) = \tilde{J}_3(\alpha_n)\tilde{J}_4(\beta_m) \qquad (6b)$$

Note that (6a,b) are the Fourier transforms of

$$J_{z1}(x,z) = J_1(x)J_2(z)$$

$$J_{x1}(x,z) = J_3(x)J_4(z)$$

Where the functional forms of J_1 , J_2 , J_3 , J_4 , are given in Figure 2. The coordinate forms of the current distributions shown in Figure 2 are the following:

$$J_1(x) = \frac{1}{2w}\left[1 + \left|\frac{x}{w}\right|^3\right] \qquad (7a)$$

$$J_2(z) = \frac{1}{l}\cos\left(\frac{\pi z}{2l}\right) \qquad (7b)$$

$$J_3(x) = \frac{1}{w}\sin\left(\frac{\pi x}{w}\right) \qquad (7c)$$

$$J_4(z) = \frac{z}{2l^2} \qquad (7d)$$

Taking the Fourier transform of (7) we get :

$$\tilde{J}_1(\alpha_n) = \frac{2sin(\alpha_n w)}{\alpha_n w} + \frac{3}{(\alpha_n w)^2}\left[\cos(\alpha_n w) - \frac{2sin(\alpha_n w)}{\alpha_n w} + \frac{2\{1-\cos(\alpha_n w)\}}{(\alpha_n w)^2}\right] \qquad (8a)$$

$$\tilde{J}_2(\beta_m) = \frac{\pi\cos(\beta l)}{\left(\frac{\pi}{2}\right)^2 - (\beta l)^2} \qquad (8b)$$

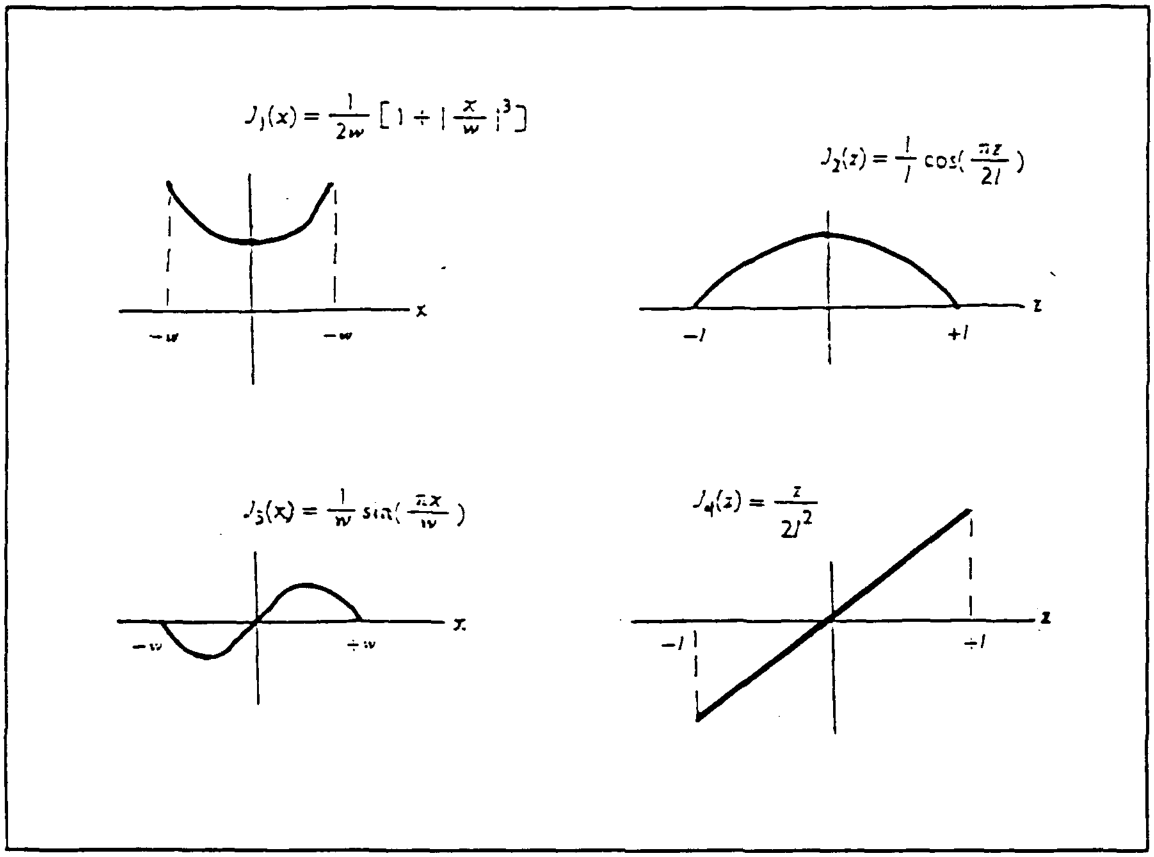

Figure 2. Forms of assumed current distributions.

$$\tilde{J}_3(\alpha_n) = \frac{2\pi \sin(\alpha_n w)}{(\alpha_n w)^2 - \pi^2} \tag{8c}$$

$$\tilde{J}_4(\beta_m) = \frac{\cos(\beta l)}{\beta l} - \frac{\sin(\beta l)}{(\beta l)^2} \tag{8d}$$

Corresponding to the structure in Figure 1, w in (7) and (8) is the half width of the strip and l is the half length of the strip.

2. Procedure of solution for resonant frequency

The field equations (1a,b)

$$\tilde{Z}_{zz}\tilde{J}_z + \tilde{Z}_{zx}\tilde{J}_x = \tilde{E}_z$$

$$\tilde{Z}_{xz}\tilde{J}_z + \tilde{Z}_{xx}\tilde{J}_x = \tilde{E}_x$$

were derived assuming the existence of J_x , and any solution derived from (1a,b) must satisfy the equations (1a) and (1b) simultaneously. Since there are four unknowns, c_j , d_k , $\tilde{E}_z$, $\tilde{E}_x$ in (1a,b), they can not be solved in general. We introduce Galerkin's procedure by taking inner products of (1a) with $\tilde{J}_{zl}$ and likewise (1b) with $\tilde{J}_{xl}$. Then, apply the (3a,b),

$$c_j \sum_{\alpha,\beta} \tilde{J}_{zj}\tilde{Z}_{zz}\tilde{J}_{zi} + d_k \sum_{\alpha,\beta} \tilde{J}_{xk}\tilde{Z}_{zx}\tilde{J}_{zi} = \sum_{\alpha,\beta} \tilde{E}_z\tilde{J}_{zi} \tag{9a}$$

$$c_j \sum_{\alpha,\beta} \tilde{J}_{zj}\tilde{Z}_{xz}\tilde{J}_{xi} + d_k \sum_{\alpha,\beta} \tilde{J}_{xk}\tilde{Z}_{xx}\tilde{J}_{xi} = \sum_{\alpha,\beta} \tilde{E}_x\tilde{J}_{xi} \tag{9b}$$

By the virtue of Parseval's theorem, the right sides of (9) are zero, so we have

$$c_j \sum_{\alpha,\beta} \tilde{J}_{zj}\tilde{Z}_{zz}\tilde{J}_{zi} + d_k \sum_{\alpha,\beta} \tilde{J}_{xk}\tilde{Z}_{zx}\tilde{J}_{zi} = 0 \tag{10a}$$

$$c_j \sum_{\alpha,\beta} \tilde{J}_{zj} \tilde{Z}_{xz} \tilde{J}_{xi} + d_k \sum_{\alpha,\beta} \tilde{J}_{xk} \tilde{Z}_{xx} \tilde{J}_{xi} = 0 \tag{10b}$$

In second-order approximation, equations (10) form the matrix solution with dimensions $[2 \times 2]$. The matrix is symmetrical. In order to have nontrivial values for c_j , d_k , the determinant of the matrix must be zero.

a. First order approximation

For the lowest-order approximation, here called first-order, we assume the current only in z-direction ($\tilde{J}_x = 0$, $i = 1, j = 1$), and choose the current distribution $J_{z1} = J_1(x) \times J_2(z)$. Since we have current only in the z-direction ($\tilde{J}_z \neq 0$, $\tilde{J}_x = 0$), then from (10) there remains only the equation:

$$c_1 \sum_{\alpha,\beta} \tilde{J}_{z1} \tilde{Z}_{zz} \tilde{J}_{z1} = 0 \tag{11}$$

By using trial values of frequency, the solution f_0 (resonant frequency) is found as the value best satisfying (11)

b. Second order approximation

For the second order solution we assume current exists in both directions (z and x) and choose the current distribution,

$$\tilde{J}_{z1}(\alpha_n, \beta_m) = \tilde{J}_1(\alpha_n) \tilde{J}_2(\beta_m)$$

$$\tilde{J}_{x1}(\alpha_n, \beta_m) = \tilde{J}_3(\alpha_n) \tilde{J}_4(\beta_m)$$

Since we have current in the z and x-directions ($\tilde{J}_z \neq 0$, $\tilde{J}_x \neq 0$), then we have the matrix solution

$$c_1 \sum_{\alpha,\beta} \tilde{J}_{z1} \tilde{Z}_{zz} \tilde{J}_{z1} + d_1 \sum_{\alpha,\beta} \tilde{J}_{z1} \tilde{Z}_{zx} \tilde{J}_{x1} = 0 \qquad (12a)$$

$$c_1 \sum_{\alpha,\beta} \tilde{J}_{x1} \tilde{Z}_{xz} \tilde{J}_{z1} + d_1 \sum_{\alpha,\beta} \tilde{J}_{x1} \tilde{Z}_{xx} \tilde{J}_{x1} = 0 \qquad (12b)$$

In this case we find f_0 as the frequency for which the determinant of the coefficients vanishes.

C. METHOD OF SOLUTION FOR PROPAGATION CONSTANT

The increasing use of microstrip lines at microwave frequencies has recently created considerable interest in the study of dispersion characteristics of these lines. Until quite recently, much of the work on the microstrip line was based on a TEM analysis. This analysis is employed to calculate the static capacitance of the structure from which the characteristic impedance and the propagation wavenumber are subsequently derived. However, this analysis, which is necessarily approximate, is ineffective for estimating the dispersion properties of the line at higher frequencies [Ref. 14].

1. Field equation

The analysis technique for computing dispersion characteristics of shielded transmission line of the type considered here has been carried out by Mittra and Itoh [Ref. 3]. The formulation of the problem applied

Galerkin's method of moments in the Fourier transform domain and is summarized in a set of Green's function equations like those used above.

$$\tilde{Z}_{zz}(\alpha_n, \beta_m)\tilde{J}_z(\alpha_n) + \tilde{Z}_{zx}(\alpha_n, \beta_m)\tilde{J}_x(\alpha_n) = \tilde{E}_z(\alpha_n) \tag{13a}$$

$$\tilde{Z}_{xz}(\alpha_n, \beta_m)\tilde{J}_z(\alpha_n) + \tilde{Z}_{xx}(\alpha_n, \beta_m)\tilde{J}_x(\alpha_n) = \tilde{E}_x(\alpha_n) \tag{13b}$$

Where β is the unknown propagation constant, and $\alpha_n = (n - 1/2)\frac{\pi}{a}$. Coefficients $\tilde{Z}_{zz,}\tilde{Z}_{zx,}\tilde{Z}_{xz,}\tilde{Z}_{xx}$ are the Green's impedance functions and are defined as shown in the previous section, and

$$\tilde{J}_x(\alpha_n) = \int_{-1.6}^{1.6} \tilde{J}_x(x)e^{-j\alpha_n x}dx \tag{14a}$$

$$\tilde{J}_z(\alpha_n) = \int_{-1.6}^{1.6} \tilde{J}_z(x)e^{-j\alpha_n x}dx \tag{14b}$$

$\tilde{J}_z$ and $\tilde{J}_x$ are the transforms of strip currents J_z and J_x. Notice that $\tilde{E}_z$ and $\tilde{E}_x$ are unknown since the electric fields $E_z(x,D + T)$ and $E_x(x,D + T)$ are unknown for $w < |x| < a$, though they are zero on the strip. The field equations in (13a,b) are similar to the field equations in (1a,b), but with current distributions assumed uniform in the z-direction and propagation constant β is now the unknown.

2. **Procedure of solution for propagation constant**

The first step is to expand the unknowns $\tilde{J}_z$ and $\tilde{J}_x$ in terms of known basis functions $\tilde{J}_{zj}$ and $\tilde{J}_{xk}$

$$\tilde{J}_z(\alpha_n) = \sum_{j=1}^{N2} c_j \tilde{J}_{zj}(\alpha_n) \tag{15a}$$

$$\tilde{J}_x(\alpha_n) = \sum_{k=1}^{N1} d_k \tilde{J}_{xk}(\alpha_n) \tag{15b}$$

Where c_j and d_k are unknown coefficients. The basis functions $\tilde{J}_{xk}$ and $\tilde{J}_{zj}$ must be chosen such that their inverse Fourier transforms are nonzero only on the strip $|x| < w$. After substituting (15) into (13) and taking inner products with the basis functions $\tilde{J}_{zi}$ and $\tilde{J}_{xi}$ for different values of i. This process yields the equation,

$$\sum_{j=1}^{N1} K_{ij}^{(1,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(1,2)} d_k = 0 \quad , i = 1,2,3,..,N2. \tag{16a}$$

$$\sum_{j=1}^{N1} K_{ij}^{(2,1)} c_j + \sum_{k=1}^{N2} K_{ik}^{(2,2)} d_k = 0 \quad , i = 1,2,3,..,N1. \tag{16b}$$

Where from definition of inner products associated with the Fourier transform defined by (14), the matrix elements are

$$K_{ij}^{(1,1)}(\beta) = c_j \sum_{n=-\infty}^{+\infty} \tilde{J}_{zi}(\alpha_n)\tilde{Z}_{zz}(\alpha_n,\beta_m)\tilde{J}_{zj}(\alpha_n) \tag{17a}$$

$$K_{ik}^{(1,2)}(\beta) = d_k \sum_{n=-\infty}^{+\infty} \tilde{J}_{zi}(\alpha_n)\tilde{Z}_{zx}(\alpha_n,\beta_m)\tilde{J}_{xk}(\alpha_n) \tag{17b}$$

$$K_{ij}^{(2,1)}(\beta) = c_j \sum_{n=-\infty}^{+\infty} \tilde{J}_{xi}(\alpha_n)\tilde{Z}_{xz}(\alpha_n,\beta_m)\tilde{J}_{zj}(\alpha_n) \tag{17c}$$

$$K_{ik}^{(2,2)}(\beta) = d_k \sum_{n=-\infty}^{+\infty} \tilde{J}_{xi}(\alpha_n)\tilde{Z}_{xx}(\alpha_n,\beta_m)\tilde{J}_{xk}(\alpha_n) \tag{17d}$$

The right hand side of (13) can be eliminated in Galerkin's process via the application of Parseval's relation.

Now the simultaneous equations (16) are solved for the propagation constant β at each frequency ω by setting the determinant of the coefficient matrix equal to zero.

a. First order approximation.

We choose of matrix size : N1 = 0, N2 = 1. The matrix problem reduces to :

$$c_1 \sum_{\alpha,\beta} \tilde{J}_{z1}(\alpha)\tilde{Z}_{zz}(\alpha,\beta)\tilde{J}_{z1}(\alpha) = 0 \tag{18}$$

b. Second order approximation.

We choose matrix of size : N1 = 1, N2 = 1. The matrix to be solved is

$$c_1 \sum_{\alpha, \beta} \tilde{J}_{z1}(\alpha) \tilde{Z}_{zz}(\alpha, \beta) \tilde{J}_{z1}(\alpha) + d_1 \sum_{\alpha, \beta} \tilde{J}_{z1}(\alpha) \tilde{Z}_{zx}(\alpha, \beta) \tilde{J}_{x1}(\alpha) = 0 \tag{19a}$$

$$c_1 \sum_{\alpha, \beta} \tilde{J}_{x1}(\alpha) \tilde{Z}_{xz}(\alpha, \beta) \tilde{J}_{z1}(\alpha) + d_1 \sum_{\alpha, \beta} \tilde{J}_{x1}(\alpha) \tilde{Z}_{xx}(\alpha, \beta) \tilde{J}_{x1}(\alpha) = 0 \tag{19b}$$

Typical output values for the moment-method calculation of propagation constant β for various frequencies are shown in Table 3 and Table 4 on page 24, Figure 6 on page 29 and Figure 7 on page 30.

D. METHOD OF SOLUTION FOR FRINGING CAPACITANCE

In this work, the resonant frequency data of the suspended substrate resonators have been used for estimating the end effect at the open ends of suspended substrate lines using the full-wave theory. In order to get the value of fringing capacitance, we use two methods, the equivalent length extension model and the transmission line circuit model.

The descriptions of each method are as follows :

1. Equivalent length extension model

From the dispersion relation the guide wavelength $\lambda_g = 2\frac{\pi}{\beta}$ is derived at the resonant frequency f_o of the microstrip resonator of length l. Consider the open-circuited resonator whose structure is identical to Figure 1 except that the length is l' instead of l. The length l' is determined from the resonant condition of the open circuited line $l' = \frac{\lambda_g}{2}$.

The hypothetical extension of the suspended substrate line, which accounts for the end effect, is given by $\Delta l = l' - l$. In terms of TEM model for transmission line characteristic impedance is defined by L (Henry/meter) and C_o (Farad/meter), and we have equations:

$$Z_o = \sqrt{L/C_o} \qquad (20a)$$

$$\beta = \omega\sqrt{LC_o} \qquad (20b)$$

From equations (20a) and (20b), the capacitance C_o is defined

$$C_o = \frac{\beta}{Z_o \omega_o} \qquad (21)$$

where C_o is capacitance/meter of line. The value of fringing capacitance ΔC is

$$\Delta C = \Delta l C_o = (l' - l)\frac{\beta}{Z_o \omega_o} \qquad (22)$$

In this work we calculate the end effect for a single open end, as shown in Figure 3 (for $\frac{\Delta L}{2}$) using the computer program in Appendix D.

2. Transmission line circuit model

The value of fringing capacitance can be calculated by using the approach of the transmission circuit model. The input impedance seen from the center of the strip resonator is calculated using formula as the following [Ref. 12]:

$$Z_{in} = \frac{Z_c + jZ_o \tan(\beta l)}{Z_o + jZ_c \tan(\beta l)} Z_o \approx 0.0 \qquad (23)$$

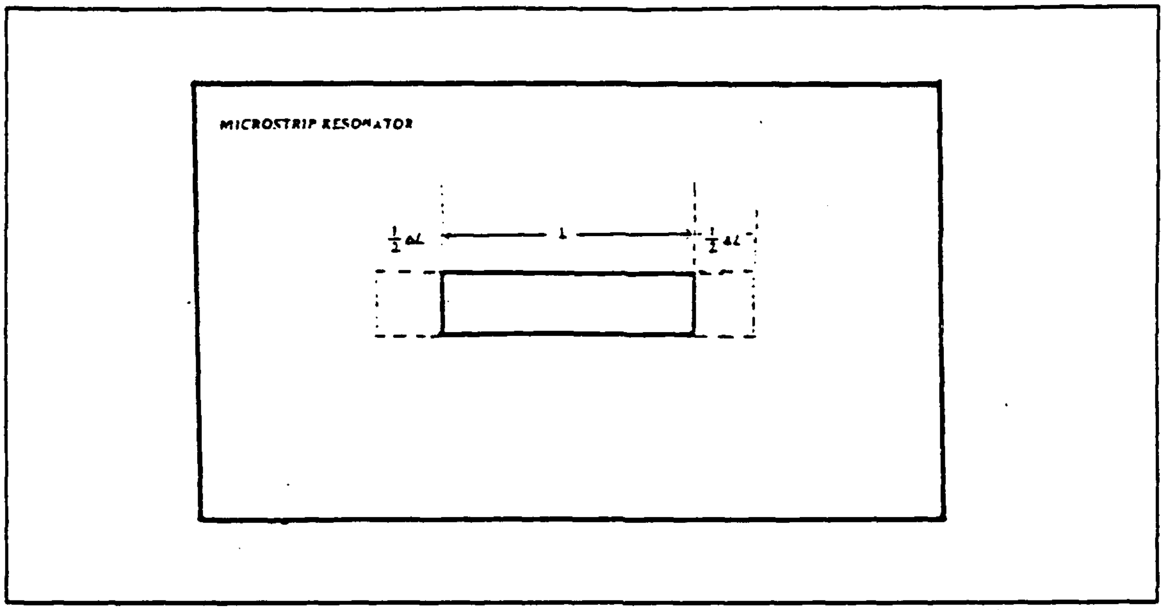

Figure 3. End effect on substrate resonator

Where

$$Z_c = \frac{1}{j\omega_o \Delta C}$$

and ΔC is a fringing capacitance, l is the half of strip length, β is the propagation constant, obtained as explain in the previous section, and Z_o is the characteristic impedance of strip. The formula to be used to calculate characteristic impedance Z_o is the following [Ref. 17]:

For $0 < W < a$

$$Z_o = \frac{z_1}{\sqrt{\varepsilon_{eff}}}$$

$$Z_1 = 60[V + R\, l_n\{6G/W + \sqrt{1 + 4(G/W)^2}\,\}]$$

Where,

$$V = -1.7866 - 0.2035(T/G) + 0.4750(2A/G)$$

$$R = 1.0835 + 0.1007(T/G) - 0.09457(2A/G)$$

$$\sqrt{\varepsilon_{eff}} = \frac{\beta c}{\omega}$$

and c is the free-space velocity of light. This formula has been confirmed by variational calculation [Ref. 19], and Z_o is assumed here to vary only slowly with frequency. The computer program for this calculation is shown in Appendix D. In Table 2 is shown the values of Z_o, the characteristic impedance for different value of ε_r and the width of strip .

Table 2. THE CHARACTERISTIC IMPEDANCE OF SUSPENDED SUBSTRATE LINE

Dielectric constant (ε_r)	Width of strip (mm)	Impedance first order (β), Z_o	Impedance second order (β), Z_o
2.2	0.711	94.635	94.220
	1.0	79.185	78.789
	1.25	68.963	68.596
9.8	0.711	70.651	70.204
	1.0	60.533	59.916
	1.25	53.446	52.732

The true value of fringing capacitance ΔC can be founded by using trial values in (23) and looking for the minimum value of the input impedance Z_{in}.

III. DISCUSSION OF RESONATOR COMPUTATION

A. INTRODUCTION

The calculation of the *suspended stripline* resonator in this work used numerical computations in a scale model of millimeter wave integrated circuits. The numerical results presented in this paper have been generated on a main frame IBM 370 using FORTRAN 77 and compiler VS 2 FORTRAN, since this compiler is 10 times faster than WATFOR 77. The solution for resonant frequencies and propagation constant was made by taking the absolute value of the determinant matrix. This approach leads to the minimum value for the system determinant rather than crossing the axis of determinant = 0. All output data together are shown in Appendix A, while the complete data values are shown in Table 3 for ε_r = 2.2 and Table 4 for ε_r = 9.8. The computation results for fringing capacitance using the transmission line circuit model agree with the computation result using the length extension model, as is shown by the very small difference between results of these two methods.

For each of these methods we can see that as the width of strip increases the resonant frequency of a resonator of given length increases as shown in Figure 4 and Figure 5. The complex dispersion behavior of this line system is shown by the decrease of β with increasing the width of strip. Also the value of fringing capacitance goes up with width. This situation may be due to the fact that the end region will store more accumulated charge with wider lines of the strip. There will be currents flowing in the end region, corresponding to the extra charge [Ref. 6].

Table 3. THE DATA RESULTS FOR ε_r = 2.2

Method of solution	Width of strip, w (mm)	Resonant frequency, f_o (GHZ)	Propagation constant, β (Rad/m)	End-effect model, ΔC_1 (Farad)	Transmission line model, ΔC_2 (Farad)	Error, ΔC_{12}
First order	0.711	12.9096	301.8001	0.8050E-14	0.8060E-14	1.E-17
	1.0	13.0237	301.0026	1.0152E-14	1.0167E-14	1.E-17
	1.25	13.0913	300.5175	1.2024E-14	1.2043E-14	2.E-17
Second order	0.711	12.8582	301.9217	0.8038E-14	0.8048E-14	1.E-17
	1.0	12.9668	301.1930	1.0099E-14	1.0114E-14	1.E-17
	1.25	13.0331	300.7837	1.1906E-14	1.1923E-14	2.E-17

Table 4. THE DATA RESULTS FOR ε_r = 9.8

Method of solution	Width of strip, w (mm)	Resonant frequency, f_o (GHZ)	Propagation constant, β (Rad/m)	End-effect model, ΔC_1 (Farad)	Transmission line model, ΔC_2 (Farad)	Error, ΔC_{12}
First order	0.711	9.4069	294.5652	2.3461E-14	2.3537E-14	8.E-17
	1.0	9.7084	293.5173	2.7951E-14	2.8050E-14	10E-17
	1.25	9.8917	292.9945	3.1858E-14	3.1977E-14	1.E-16
Second order	0.711	9.35427	294.7848	2.3477E-14	2.3551E-14	7.E-17
	1.0	9.6251	293.9983	2.7820E-14	2.7915E-14	9.E-17
	1.25	9.7804	293.6181	3.1695E-14	3.1806E-14	1.E-16

B. RESONANT FREQUENCY

For the first order approximation the modified summation in terms of α and β to (11) is represented as

$$\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} G(\alpha_n, \beta_m) = 0 \tag{24}$$

Where $G(\alpha_n, \beta_m)$ is the product form of basis functions and Green's impedance function. The computer programs for first and second order approximation for J_z is shown in Appendix B. These programs calculate the value of the determinant with various values of strip width in order to get the resonant frequency, with dielectric constant for substrate : $\varepsilon_r = 2.2$ for duroid, and $\varepsilon_r = 9.8$ for alumina. The solutions for first order are found :

$$\sum_{n=-30}^{+30} \sum_{m=-3000}^{+3000} \tilde{J}_{z1}^{2}(\alpha_n, \beta_m) \tilde{Z}_{zz}(\alpha_n, \beta_m) = 0 \tag{25}$$

In the x-direction we take the sum from from n=-30 to n= +30 and in the z-direction from m=-3000 to m= +3000. Since a higher limit does not change the result as shown in Table 5.

In a similar way, we may compute the second order approximation with the same summation limits for α_n and for β_m. All data outputs for resonant frequency appear in Appendix A.

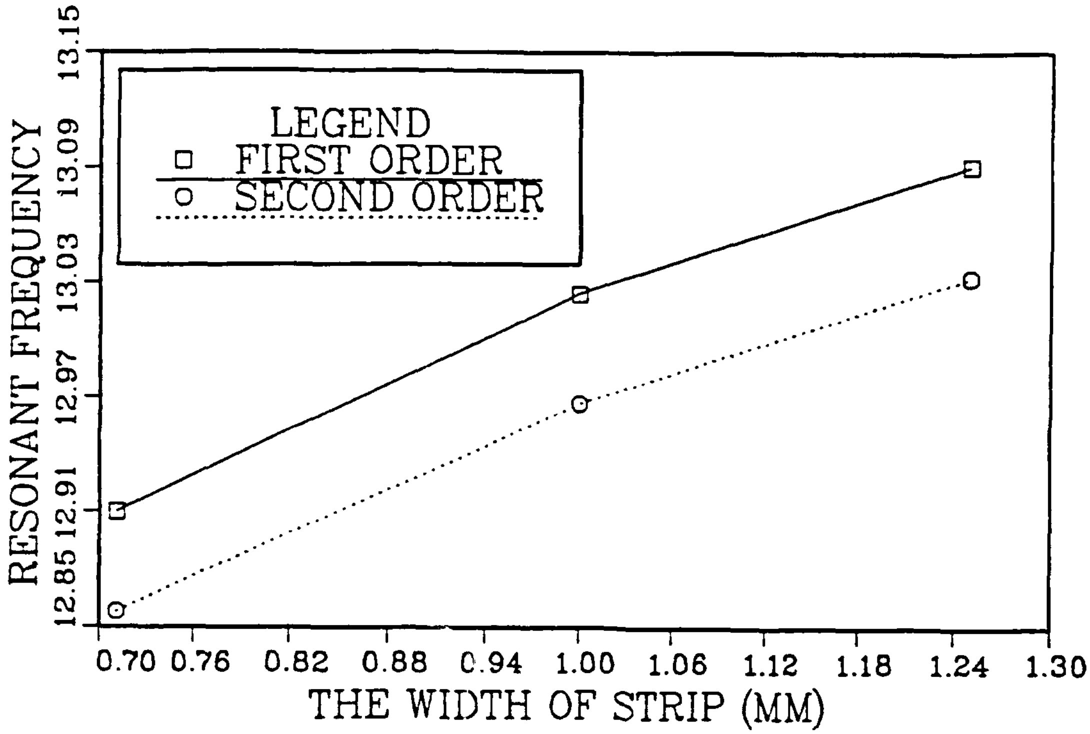

Figure 4. Resonant frequency versus width of strip for ε_r = 2.2

Table 5. THE COMPARISON FOR RESONANT FREQUENCY IN FIRST ORDER APPROXIMATION

α_n	β_m	ε_r	w	Resonant Frequency (GHZ)
n = -30 to n = +30	m = -3000 to m = +3000	2.2	1.0	13.023998
n = -40 to n = +40	m = -4000 to m = +4000	2.2	1.0	13.023498

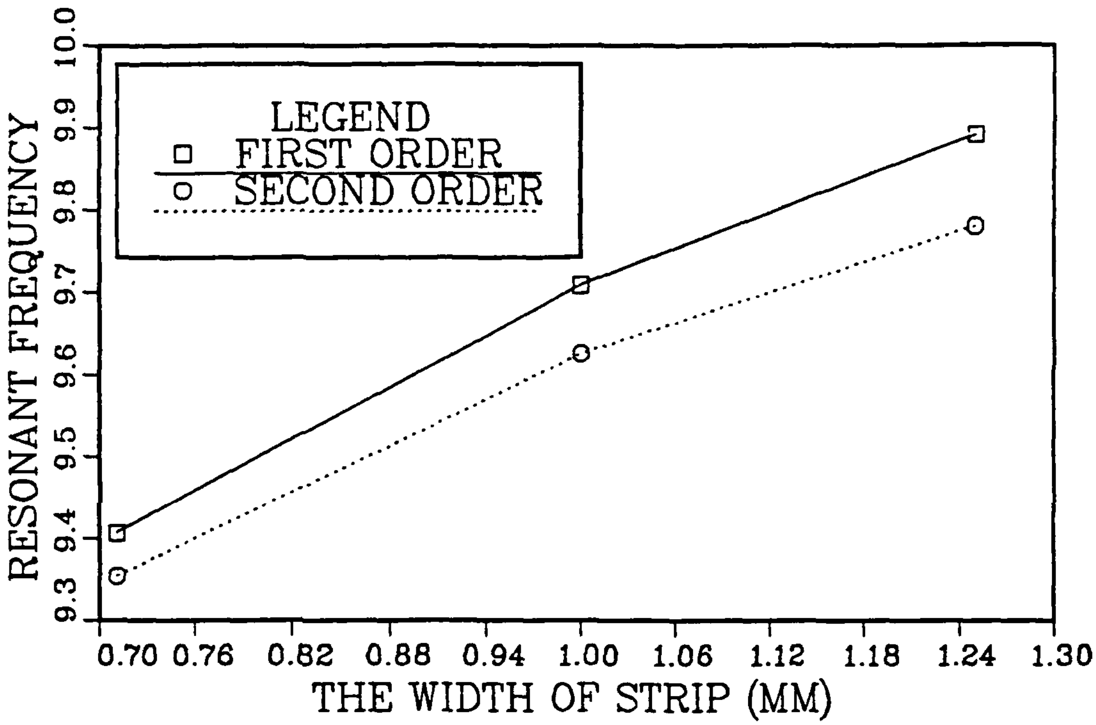

Figure 5. *Resonant frequency versus width of strip for $\varepsilon_r = 9.8$*

C. PROPAGATION CONSTANT

The program to be used for computation of the propagation constant is shown in Appendix C. An initial approximation is taken as the following [Ref. 7]:

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_r + 1}{2}} \tag{26}$$

Where c is the free-space velocity of light. To find the exact value of propagation constant, we sweep the value of β from 1% to 200% of (26) in order to bring the value of the matrix determinant in (18) to zero. The propagation constant can be identified by the occurrence of a minimum value of the matrix determinant. In this case we take the summation limits from n=-40 to n=+40, since a higher limit does not change the result as shown in Table 6.

Table 6. THE COMPARISON FOR PROPAGATION CONSTANT IN FIRST ORDER APPROXIMATION

α_n	ε_r	w	Propagation Constant (β)
n=-40 to n=+40	2.2	1.0	301.80
n=-100 to n=+100	2.2	1.0	301.81

(See in Appendix A for the complete results)

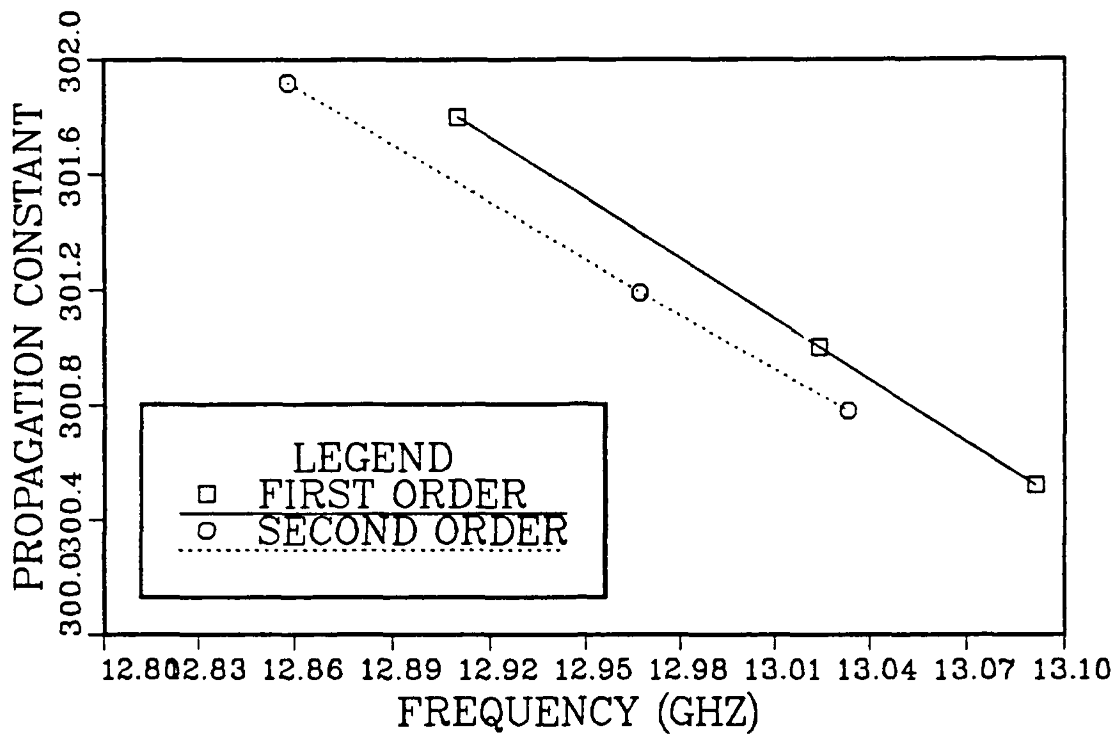

Figure 6. Resonant frequency versus propagation constant for $\varepsilon_r = 2.2$

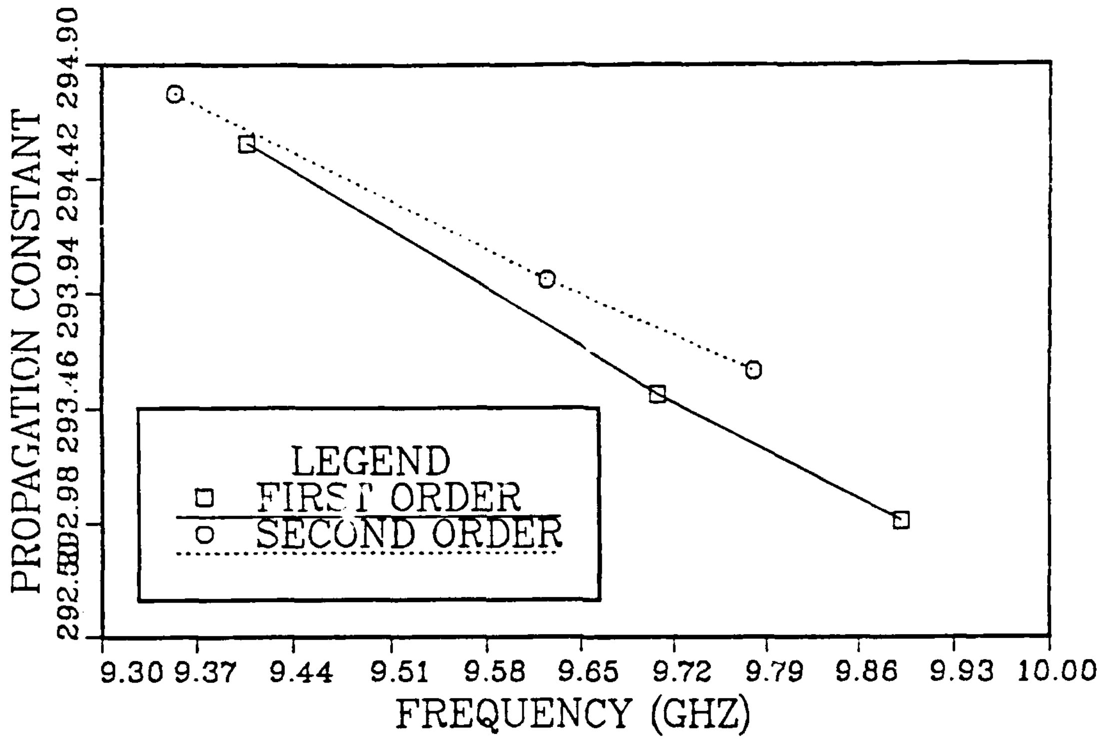

Figure 7. Resonant frequency versus propagation constant for $\varepsilon_r = 9.8$

D. PREDICTION OF FRINGING CAPACITANCE VALUE

The value of fringing capacitance can be found by solving equation (22) for the equivalent length extension model and solving equation (23) for for transmission line circuit model. The computer program used to calculate the fringing capacitance are shown in Appendix D. From the results in Table 3 and Table 4 we see that these two methods agree since there is only a small difference in their results.

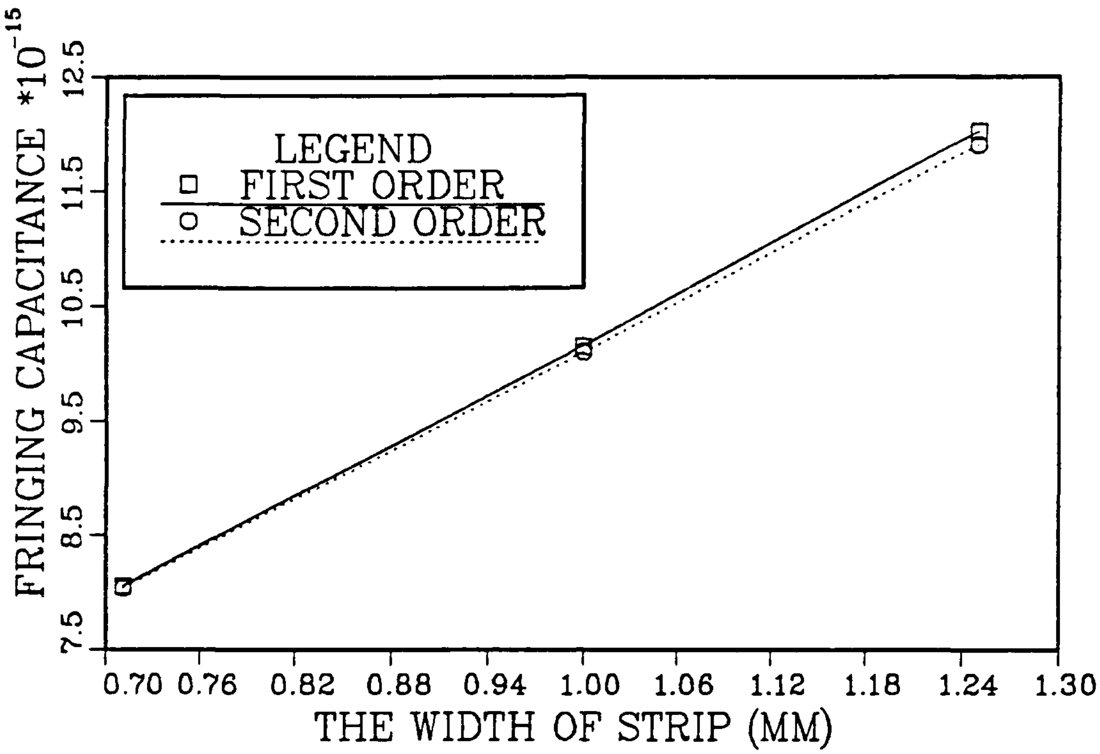

Figure 8. Fringing capacitance versus width of strip for $\varepsilon_r = 2.2$

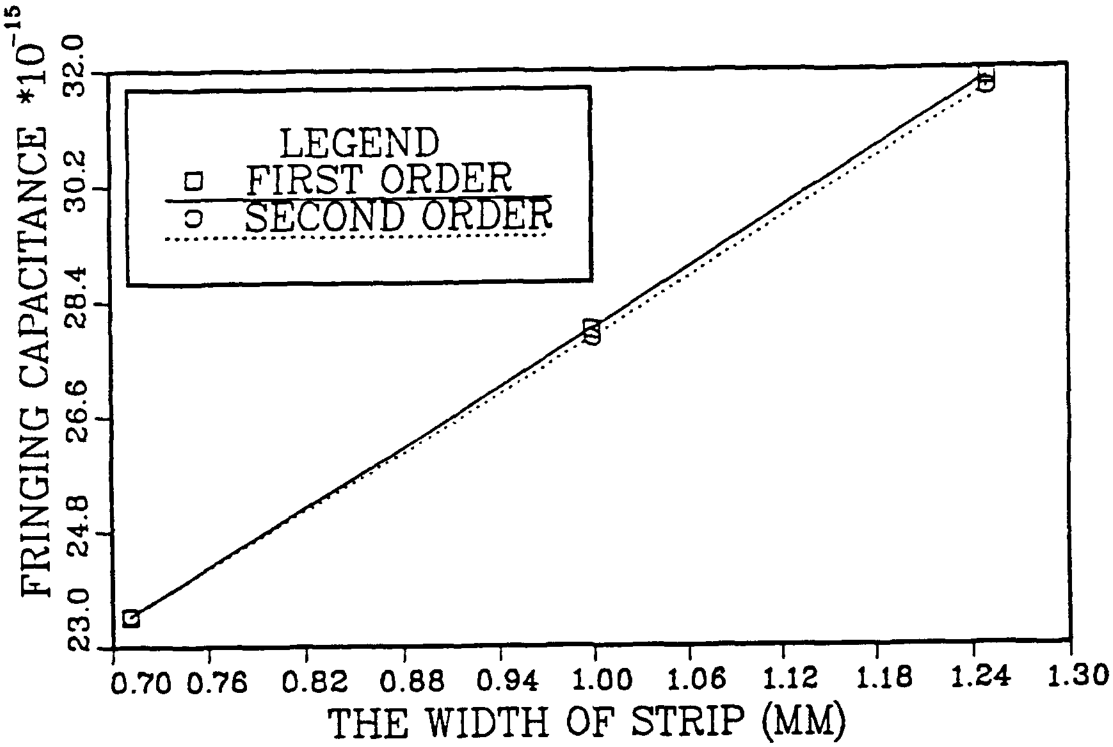

Figure 9. **Fringing capacitance versus width of strip for ε_r = 9.8**

IV. CONCLUSION

From a computational point of view, the calculation using Green's function is very effective to solve the stripline resonator problem. Actually the computed result for the resonant frequency depends on the choice of the definition of the current distributions in the x-direction and the z-direction. In the Table 3, Table 4, Figure 5 and Figure 6 it is seen that the resonant frequencies increases as the width of strip increases. We do not see this phenomenon in a resonator without suspended substrate, as shown in [Ref. 1].

The values of the fringing capacitance found here appear to be reasonable: the values are of the order of 10^{-14} Farad. The error between the two methods of calculation is very small, providing a confirmation of the length extension model.

APPENDIX A. OUTPUT DATA

A. RESONANT FREQUENCY

1. First order approximation

- For name w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	DETERMINANT'S VALUES
12.909532500000	0.0195708908
12.909547800000	0.0134981312
12.909563100000	0.0063938200
12.909578300000	0.0007518493
12.909593600000	0.0081374571

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	DETERMINANT'S VALUES
13.023622500000	0.0549158342
13.023684500000	0.0306886621
13.023746500000	0.0076220483
13.023808500000	0.0151751861
13.023870500000	0.0382801555

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY (GHZ)	DETERMINANT'S VALUES
13.090996700000	0.0922133327
13.091095900000	0.0595099851
13.091195100000	0.0294705480
13.091294300000	0.0022094622
13.091393500000	0.0342780240
13.091492700000	0.0658574104
13.091591800000	0.0986036658

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY (GHZ)	DETERMINANT'S VALUES
9.406851964275	0.0055711373
9.406858630475	0.0026056429
9.406865296675	0.0003598495
9.406871962875	0.0033253400
9.406878629075	0.0062908285

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY (GHZ)	DETERMINANT'S VALUES
9.708079097500	0.1101123587
9.708178995000	0.0734216453
9.708278892500	0.0367312736
9.708378790000	0.0000412434
9.708478687500	0.0366484453
9.708578585000	0.0733377923
9.708678482500	0.1100267979

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	DETERMINANT'S VALUES
9.891719662500	0.0013436906
9.891720994500	0.0009188024
9.891722326500	0.0004939143
9.891723658500	0.0000690263
9.891724990500	0.0003558617
9.891726322500	0.0007807497
9.891727654500	0.0012056375

2. Second order approximation

- For w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	THE VALUES OF DETERMINANT
12.858237475000	134.7679778482
12.858239430000	45.1968072590
12.858241385000	44.3743231685
12.858243340000	133.9454133199

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	THE VALUES OF DETERMINANT
12.966809460000	21.9744171435
12.966809956000	12.5409965103
12.966810452000	3.1075768752
12.966810948000	6.3258416755
12.966811444000	15.7592591259

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY(GHZ)	THE VALUES OF DETERMINANT
13.033089704000	95.3389230655
13.033102202000	36.4297524373
13.033114700000	168.1980546933

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY(GHZ)	THE VALUES OF DETERMINANT
9.354219440000	838.0000000000
9.354244230000	369.8750000000
9.354269030000	96.0000000000
9.354293820000	605.0000000000

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY(GHZ)	THE VALUES OF DETERMINANT
9.624875000000	2076.6096794134
9.625000000000	1067.1692908816
9.625125000000	57.7672394809
9.625250000000	951.5964768065
9.625375000000	1960.9218599920

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	THE VALUES OF DETERMINANT
9.780300000000	359.7268520610
9.780360000000	83.7447038466
9.780420000000	192.2324952445
9.780480000000	468.2047453591

B. PROPAGATION CONSTANT

1. First order approximation

- For w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
12.909578300	301.793213000	0.006551314
12.909578300	301.796631000	0.003314178
12.909578300	301.800049000	0.000016721
12.909578300	301.803467000	0.003176749
12.909578300	301.806885000	0.006300699

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
13.023746500	300.933559227	0.043851649
13.023746500	300.968062043	0.016535467
13.023746500	301.002564859	0.010783630
13.023746500	301.037067675	0.038105642
13.023746500	301.071570490	0.065430568

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
13.091294300	300.448130204	0.047616359
13.091294300	300.482811969	0.023635390
13.091294300	300.517493734	0.000348146
13.091294300	300.552175499	0.024334248
13.091294300	300.586857264	0.048322918

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.406865000	294.290520504	0.218513360
9.406865000	294.427868289	0.122327343
9.406865000	294.565216074	0.026102604
9.406865000	294.702563859	0.070160849
9.406865000	294.839911643	0.166463009

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.708379000	293.233772058	0.208247936
9.708379000	293.375522190	0.123411162
9.708379000	293.517272321	0.038539359
9.708379000	293.659022453	0.046367465
9.708379000	293.800772584	0.131309303

9.708379000	293.942522716	0.216286146

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.891724000	292.705621320	0.146004764
9.891724000	292.850048436	0.069357733
9.891724000	292.994475552	0.007321405
9.891724000	293.138902668	0.084032642
9.891724000	293.283329783	0.160775971

2. Second order approximation

- For w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
12.858241000	301.914079040	264.8257310433
12.858241000	301.916633866	188.2549021972
12.858241000	301.919188693	111.6834688237
12.858241000	301.921743520	35.1114309231
12.858241000	301.924298346	41.4612114998
12.858241000	301.926853173	118.0344584496
12.858241000	301.929407999	194.6083099229
12.858241000	301.931962826	271.1827659171

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
12.966810000	301.189557419	47.3499570198
12.966810000	301.191275018	25.5508907438
12.966810000	301.192992616	3.7517095664
12.966810000	301.194710215	18.0475865116
12.966810000	301.196427814	39.8469974898

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
13.033102000	300.776834261	27.3862465221
13.033102000	300.778560641	15.0664900067
13.033102000	300.780287021	2.7466685987
13.033102000	300.782013401	9.5732177036
13.033102000	300.783739781	21.8931688989

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.354269000	294.739297051	402.7507924038
9.354269000	294.762060358	192.4514179721

9.354269000	294.784823665	17.8618972464
9.354269000	294.807586971	228.1891527630
9.354269000	294.830350278	438.5303480889

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.625125000	293.904601953	311.5213476632
9.625125000	293.951446805	120.8819158085
9.625125000	293.998291657	69.7832174509
9.625125000	294.045136509	260.4740500634
9.625125000	294.091981361	451.1905799777

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	BETA (RAD/METER)	DETERMINANT
9.780360000	293.532445612	204.6350496775
9.780360000	293.561005835	136.5694649559
9.780360000	293.589566057	68.4983401777
9.780360000	293.618126279	0.4216756325
9.780360000	293 646686502	67.6605283907
9.780360000	293.675246724	135.7482716025
9.780360000	293.703806947	203.8415537136

C. FRINGING CAPACITANCE USING LENGTH EXTENSION MODEL

1. First order approximation

- For w = 0.711 mm and ε_r = 2.2

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
12.9095780	301.800050	0.00000000000000805039

- For w = 1.0 mm and ε_r = 2.2

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
13.0237470	301.002570	0.000000000000001015215

- For w = 1.25 mm and ε_r = 2.2

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
13.0912940	300.517490	0.0000000000000120243

- For w = 0.711 mm and ε_r = 9.8

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.4068650	294.565220	0.000000000000023461

- For w = 1.0 mm and ε_r = 9.8

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.7083790	293.517270	0.000000000000027951

- For w = 1.25 mm and ε_r = 9.8

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.8917240	292.994480	0.000000000000003185787

2. Second order approximation

- For $w = 0.711$ mm and $\varepsilon_r = 2.2$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
12.8582410	301.921740	0.000000000000080382

- For $w = 1.0$ mm and $\varepsilon_r = 2.2$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
12.9668100	301.192990	0.00000000000010099657

- For $w = 1.25$ mm and $\varepsilon_r = 2.2$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
13.0331020	300.783740	0.000000000000119056853476236

- For $w = 0.711$ mm and $\varepsilon_r = 9.8$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.3542690	294.784800	0.000000000000234773

- For $w = 1.0$ mm and $\varepsilon_r = 9.8$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.6251250	293.998290	0.000000000000278197027432552

- For $w = 1.25$ mm and $\varepsilon_r = 9.8$

RESONANT FREQUENCY	PROPAGATION CONSTANT	FRINGING CAPACITANCE (FARAAD)
9.7803600	293.618130	0.000000000000316946

D. FRINGING CAPACITANCE USING TRANSMISSION LINE CIRCUIT MODEL

1. First order approximation

- For w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
12.9095783	0.0000000000000800	0.043897341449
12.9095783	0.0000000000000802	0.029423400665
12.9095783	0.0000000000000804	0.014949733818
12.909578?	0.0000000000000806	0.000476340899
12.9095783	0.0000000000000808	0.013996778098
12.9095783	0.0000000000000810	0.028469623184
12.9095783	0.0000000000000812	0.042942194364

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
13.0237470	0.0000000000000101667	0.000020988378
13.0237470	0.0000000000000101668	0.000010770731
13.0237470	0.0000000000000101668	0.000000553083
13.0237470	0.0000000000000101668	0.000009664563
13.0237470	0.0000000000000101668	0.000019882210
13.0237470	0.0000000000000101668	0.000030099857
13.0237470	0.0000000000000101669	0.000040317503

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
13.0912940	0.000000000000012043	0.0000123459405
13.0912940	0.000000000000012043	0.0000045583584
13.0912940	0.000000000000012043	0.0000032292234
13.0912940	0.000000000000012043	0.0000110168053
13.0912940	0.000000000000012043	0.0000188043870

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.4068650	0.0000000000002353658	0.000004112742
9.4068650	0.0000000000002353658	0.000002943926
9.4068650	0.0000000000002353658	0.000001775109
9.4068650	0.0000000000002353659	0.000000606293
9.4068650	0.0000000000002353659	0.000000562522
9.4068650	0.0000000000002353660	0.000001731339
9.4068650	0.0000000000002353660	0.000002900155

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.7083790	0.000000000000028040	0.002439262912
9.7083790	0.000000000000028045	0.001333532326
9.7083790	0.000000000000028050	0.000227305924
9.7083790	0.000000000000028055	0.000877916293
9.7083790	0.000000000000028060	0.001983634327
9.7083790	0.000000000000028065	0.003089348177

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.8917240	0.000000000000031975	0.000409506598
9.8917240	0.000000000000031976	0.000233952816
9.8917240	0.000000000000031977	0.000058399157
9.8917240	0.000000000000031978	0.000117154378
9.8917240	0.000000000000031979	0.000292707792

2. Second order approximation

- For w = 0.711 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
12.8582410	0.000000000000008044	0.003037869907
12.8582410	0.000000000000008046	0.001608804997
12.8582410	0.000000000000008048	0.000179742742
12.8582410	0.000000000000008050	0.001249316856
12.8582410	0.000000000000008052	0.002678373799

- For w = 1.0 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
12.9668100	0.000000000000010111	0.001425888209
12.9668100	0.000000000000010112	0.000922250353
12.9668100	0.000000000000010113	0.000418612916
12.9668100	0.000000000000010114	0.000085024103
12.9668100	0.000000000000010115	0.000588660705
12.9668100	0.000000000000010116	0.001092296888

- For w = 1.25 mm and ε_r = 2.2

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
13.0331020	0.0000000000000001192344	0.000010349874
13.0331020	0.0000000000000001192345	0.000006513849
13.0331020	0.0000000000000001192346	0.000002677825
13.0331020	0.0000000000000001192347	0.000001158198
13.0331020	0.0000000000000001192348	0.000004994223
13.0331020	0.0000000000000001192349	0.000008830247
13.0331020	0.0000000000000001192350	0.000011666272

- For w = 0.711 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.3542690	0.0000000000000235503	0.000205384374
9.3542690	0.0000000000000235506	0.000119294322
9.3542690	0.0000000000000235509	0.000033204290
9.3542690	0.0000000000000235512	0.000052885720
9.3542690	0.0000000000000235515	0.000138975711

- For w = 1.0 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.6251250	0.0000000000000279130	0.000283333299
9.6251250	0.0000000000000279135	0.000175879548
9.6251250	0.0000000000000279140	0.000068425835
9.6251250	0.0000000000000279145	0.000039027837
9.6251250	0.0000000000000279150	0.000146481471
9.6251250	0.0000000000000279155	0.000253935066
9.6251250	0.0000000000000279160	0.000361388622

- For w = 1.25 mm and ε_r = 9.8

FREQUENCY (GHZ)	CAPACITANCE (FARAAD)	IMPEDANCE
9.7803600	0.0000000000000318055	0.000174399228
9.7803600	0.0000000000000318058	0.000123674994
9.7803600	0.0000000000000318061	0.000072950771
9.7803600	0.0000000000000318064	0.000022226558
9.7803600	0.0000000000000318067	0.000028497644
9.7803600	0.0000000000000318070	0.000079221837
9.7803600	0.0000000000000318073	0.000129946020

APPENDIX B. PROGRAMS FOR CALCULATION OF RESONANT FREQUENCY

```
*       VARIABLE DEFINITION:
*       ====================
*       I    = REPRESENT THE REGION 1,2,3 IN RESONATOR
*       ALPA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (x-direction)
*       BETA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (z-direction)
*       A    = A HALF WIDTH OF RESONATOR DEVICE = 1.6 mm
*       D    = HEIGHT OF RESONATOR BELOW SUBSTRATE = 0.66 mm
*       T    = THE THICKNESS OF SUBSTRATE = 0.255 mm
*       W    = THE WIDTH OF STRIP, CAN BE 0.711 mm, 1.0 mm, or 1.25 mm
*       H    = THE HEIGHT OF RESONATOR ABOVE SUBSTRATE = 0.66 mm
*       AL   = THE LENGTH OF STRIPLINE , ASSUME = 10 mm
*       AKI  = RADIAN FREQUENCY * SQRT OF (PERMITTIVITY*PERMEABILITY)
*       MO   = PERMEABILITY IN VACCUM = PI*4.E-7
*       MR   = PERMEABILITY IN A SUBSTRATE ASSUME = 1
*       EO   = PERMITTIVITY IN VACCUM = (1/(36*PI))*E-9 = 8.85E-12
*       ER   = PERMITIVITY IN A SUBSTRATE, CAN BE 2.2 OR 9.8
*       FREQ = FREQUENCY OPERATION IN GHZ
*       DE   = DENOMINATOR OF ZE
*       DE   = NUMERATOR OF ZE
*       DH1,DH2,DH = DENOMINATOR OF ZH
*       NH   = NUMERATOR OF ZH
*       ZE   = IMPEDANCE OF ELECTRIC FIELD, eqn (1g)
*       ZH   = IMPEDANCE OF MAGNETIC FIELD, eqn (1h)
*       ZZZ,ZZX,ZXZ,AND ZXX ARE THE GREEN'S IMPEDANCE FUNCTION (1c,d,e,f)
*       COT1 = CT1, EQN (1i)
*       COT2 = CT2, EQN (1j)
*       COT3 = CT3, EQN (1k)
*       J1   = EQN (8a)
*       J2   = EQN (8b)
*       JJ1  = Jz1   EQN (6a)
*       JJ4  = Jx1   EQN (6b)
*       JZ1  = BASIS FUNCTION OF CURRENT COMPONENT IN Z - DIRECTION
*       JX1  = BASIS FUNCTION OF CURRENT COMPONENT IN X - DIRECTION
*       K11,K12,K21,K22 = MATRIX ELEMENT
*       J1, J3 = ASSUME CURRENT DISTRIBUTION IN X - DIRECTION
*       J2, J4   = ASSUME CURRENT DISTRIBUTION IN Z - DIRECTION
*       ALL COMPUTATION IN STANDARD "MKS"
```

A. FIRST ORDER APPROXIMATION

```
*     THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE VALUE
*     OF DETERMINANT OF GREEN FUNCTION IN FIRST ORDER
*     APPROXIMATION WITH W = 0.711 mm and ER = 2.2
*     ================================================


*     MAIN PROGRAM :
*     =============

*     FUNCTION :  F1(ALPA,BETA,FREQ) = ZZZ * JZ1 **2
      COMPLEX KK,K11,F1
      INTEGER N,M,L,I
      REAL A,W,D,T,H,M0,E0,ER,PI,MAGKK,AL,FREQ,ALPA,BETA
      PARAMETER (PI = 3.141592654, E0 = 8.85E-12)
      PARAMETER (H = 0.66E-03, T = 0.255E-03, D = 0.66E-03)
      PARAMETER (M0 = PI*4.E-7, AL = 0.01, A = 1.6E-03)
      PARAMETER (ER = 2.2 , W = 0.711E-03)
*     NOTE : PARAMETER W CAN BE 0.711, 1.0 OR 1.25 mm
*     NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

      DO 15 FREQ = 12.9075,12.9125,3.34E-4
      KK = CMPLX ( 0.0,0.0 )

*     START SUMMATION IN ALPA FROM N=-30 TO N=+30
      DO 25 L = 1,61
      N = L - 31
      ALPA = (N-0.5)*PI/A
      K11 = CMPLX ( 0.0,0.0 )

*     START SUMMATION IN BETA FROM M=-3000 TO M=+3000
      DO 35 I = 1,6001
      M = I - 3001
      BETA = (M-0.5)*PI/(5.*AL)
      K11 = K11 + F1(ALPA,BETA,FREQ)
35    CONTINUE
      KK = KK + K11
25    CONTINUE
      MAGKK = CABS((KK))
      WRITE(6,100) FREQ, MAGKK
100   FORMAT(' ',3X,F25.12,3X,F35.10)
15    CONTINUE
      STOP
      END


*     FUNCTION SUBPROGRAM F1
*     ======================

      COMPLEX FUNCTION F1(ALPA,BETA,FREQ)
      COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3,GM1,GM2,GM3
      COMPLEX YE,YH,DH1,DH2,DH,ZZZ1,ZZZ2,ZZZ,JZ1
      REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
      REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,D,T,W,A,M0,PI,ER,E0
      REAL X,Y,X1,J11,J12,J13,J14,J15,J1,J21,J22,J2,JJ1
```

```
      PARAMETER (PI = 3.141592654, E0 = 8.85E-12)
      PARAMETER (H = 0.66E-03, T = 0.255E-03, D = 0.66E-03)
      PARAMETER (M0 = PI*4.E-7, AL = 0.01, A = 1.6E-03)
      PARAMETER (W = 0.711E-03, ER = 2.2 )
*     NOTE : PARAMETER W CAN BE 0.711, 1.0 OR 1.25 mm
*     NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

      YYE = -1./(2.E+9*PI*FREQ*E0)
      YE = CMPLX ( 0.0,YYE )
      YYH = 2.E+9*PI*M0*FREQ
      YH = CMPLX(0.0,YYH)
      AK1 = M0*E0*((2.E+9*PI*FREQ)**2)
      AK2 = M0*E0*ER*((2.E+9*PI*FREQ)**2)
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX(0.0,G1)
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX(0.0,COTT1)
      COT3 = CMPLX(0.0,COTT3)
      ELSE
      GM1 = CMPLX(G1,0.0)
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = 1./TANH(GMM1)
      COTT3 = 1./TANH(GMM3)
      COT1 = CMPLX(COTT1,0.0)
      COT3 = CMPLX(COTT3,0.0)
      END IF
      G2 = SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX(G2,0.0)
      GMM2 = G2*T
      COTT2 = 1./TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3 + COT1*COT3*(GM2/GM1)/ER + COT1*COT2 + (GM3/GM2)*ER
      NE = ((GM2/ER)*COT3 + GM3*COT2)*YE
      NH = GM2*COT2 + GM3*COT3
      DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
      DH2 = GM2*GM3*COT2*COT3 + GM2**2
      DH = DH1 + DH2
      ZH = ( NH / DH )*YH
      ZE = NE / DE
```

```
X1 = -1./(ALPA**2 + BETA**2)
ZZZ1 = CMPLX ( X1,0.0 )
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA*W/2.
J11 = 2.*SIN(X)/X
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = (2.*(1.-COS(X)))/(X**2)
J1 = J11 + J12*(J13 - J14 + J15)
Y = BETA*AL / 2.
J21 = PI * COS(Y)
J22 = (PI/2.)**2 - Y**2
J2 = J21 / J22
JJ1 = J1*J2
JZ1 = CMPLX(JJ1,0.0)
F1 = ZZZ*JZ1*JZ1
RETURN
END
```

B. SECOND ORDER APPROXIMATION

```
*       THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE VALUE
*       OF DETERMINANT OF GREEN FUNCTION IN SECOND ORDER
*       APPROXIMATION WITH W = 0.711 mm and ER = 2.2
*       ==================================================

*       MAIN PROGRAM :
*       =============

*       F1(ALPA,BETA,FREQ) = ZZZ * JZ1 ** 2
*       F2(ALPA,BETA,FREQ) = ZZX * JZ1 * JX1
*       F3(ALPA,BETA,FREQ) = ZXZ * JZ1 * JX1
*       F4(ALPA,BETA,FREQ) = ZXX * JX1 ** 2

        COMPLEX K11,K22,K33,K44,KK1,KK2,KK3,KK4
        COMPLEX KKK1,KKK2,KK
        COMPLEX F1,F2,F3,F4
        INTEGER M,N,I,L
        REAL MAGKK,A,W,D,T,H,M0,E0,ER,PI,AL,ALPA,BETA,FREQ
        PARAMETER ( PI=3.141592654, E0=8.85E-12 )
        PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
        PARAMETER ( M0=PI*4.E-7, AL=0.01, A =1.6E-03 )
        PARAMETER ( ER = 2.2 , W = 0.711E-03 )
*       NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
*       NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

        DO 15 FREQ = 12.8582191,12.8582687,2.48E-6
        KK1 = CMPLX( 0.0,0.0 )
        KK2 = CMPLX( 0.0,0.0 )
        KK3 = CMPLX( 0.0,0.0 )
        KK4 = CMPLX( 0.0,0.0 )

*       START SUMMATION IN ALPA FROM N=-30 TO N=+30
        DO 25 L = 1, 61
        N = L - 31
        ALPA = (N-0.5)*PI/A
        K11 = CMPLX ( 0.0,0.0 )
        K22 = CMPLX ( 0.0,0.0 )
        K33 = CMPLX ( 0.0,0.0 )
        K44 = CMPLX ( 0.0,0.0 )

*       START SUMMATION IN BETA FROM M=-3000 TO M=+3000
        DO 35 I = 1, 6001
        M = I - 3001
        BETA = (M-0.5)*PI/(5.*AL)
        K11 = K11 + F1(ALPA,BETA,FREQ)
        K22 = K22 + F2(ALPA,BETA,FREQ)
        K33 = K33 + F3(ALPA,BETA,FREQ)
        K44 = K44 + F4(ALPA,BETA,FREQ)
35      CONTINUE
        KK1 = KK1 + K11
        KK2 = KK2 + K22
        KK3 = KK3 + K33
        KK4 = KK4 + K44
25      CONTINUE
```

```
      KKK1 = KK1*KK4
      KKK2 = KK2*KK3
      KK   = KKK1 - KKK2
      MAGKK = CABS((KK))
      WRITE(6,100) FREQ, MAGKK
100   FORMAT(' ',2X,F16.12,3X,F20.10)
15    CONTINUE
      STOP
      END


*     FUNCTIONS SUBPROGRAM F1, F2, F3, F4
*     ===================================

      COMPLEX FUNCTION F1(ALPA,BETA,FREQ)
      COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
      COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
      COMPLEX ZZZ1,ZZZ2,ZZZ,JZ1
      REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
      REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,M0,PI,ER,E0
      REAL X,Y,X1,J11,J12,J13,J14,J15,J1,J21,J22,J2,JJ1
      PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12 )
      PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
      PARAMETER ( M0=PI*4.E-7, AL=0.01 )
      PARAMETER ( A=1.6E-03, W=0.711E-03 )
*     NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
*     NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

      YYE = -1./ (2.E+9*PI*FREQ*E0)
      YE = CMPLX (0.0,YYE)
      YYH = 2.E+9*PI*M0*FREQ
      YH = CMPLX (0.0,YYH)
      AK1 = M0*E0*((2.E+9*PI*FREQ)**2)
      AK2 = M0*E0*ER*((2.E+9*PI*FREQ)**2)
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX(0.0,COTT1)
      COT3 = CMPLX(0.0,COTT3)
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = 1./ TANH(GMM1)
      COTT3 = 1./ TANH(GMM3)
      COT1 = CMPLX ( COTT1,0.0 )
      COT3 = CMPLX ( COTT3,0.0 )
```

```
      END IF
      G2=SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX ( G2,0.0 )
      GMM2 = G2*T
      COTT2 = 1./ TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
      NE = ((GM2/ER)*COT3+GM3*COT2)*YE
      NH= GM2*COT2 + GM3*COT3
      DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
      DH2 = GM2*GM3*COT2*COT3 + GM2**2
      DH = DH1+DH2
      ZH = (NH / DH) * YH
      ZE = NE / DE
      X1 = -1./( ALPA**2 + BETA**2 )
      ZZZ1 = CMPLX ( X1,0.0 )
      ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
      ZZZ = ZZZ1*ZZZ2
      X = ALPA*W/2.
      Y = BETA*AL / 2.
      J11 = 2.*SIN(X)/(X)
      J12 = 3./(X**2)
      J13 = COS(X)
      J14 = J11
      J15 = (2.*(1.-COS(X))) / (X**2)
      J1 = J11+J12*(J13-J14+J15)
      J21 = PI*COS(Y)
      J22 = (PI/2.)**2 - (Y**2)
      J2 = J21/J22
      JJ1 = J1*J2
      JZ1 = CMPLX ( JJ1,0.0 )
      F1 = ZZZ*JZ1*JZ1
      RETURN
      END

      COMPLEX FUNCTION F2(ALPA,BETA,FREQ)
      COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
      COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
      COMPLEX ZZX,JZ1,JX1
      REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
      REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,M0,PI,ER,E0
      REAL X,Y,J11,J12,J13,J14,J15,J1,J21,J22,J2,JJ1
      REAL J41,J42,J4,J31,J32,J3,JJ4,ZZX1
      PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12 )
      PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
      PARAMETER ( M0=PI*4.E-7, AL=0.01 )
      PARAMETER ( A=0.16E-02, W=0.711E-03 )
*     NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
*     NOTE : PAREMETER ER CAN BE 2.2 OR 9.8
```

```
YYE = -1./(2.E+9*PI*FREQ*EO)
YE = CMPLX (0.0,YYE)
YYH = 2.E+9*PI*MO*FREQ
YH = CMPLX (0.0,YYH)
AK1 = MO*EO*((2.E+9*PI*FREQ)**2)
AK2 = MO*EO*ER*((2.E+9*PI*FREQ)**2)
AKK1 = ALPA**2 + BETA**2 - AK1
AKK2 = ALPA**2 + BETA**2 - AK2
G1 = SQRT(ABS(AKK1))
G3 = G1
IF (AKK1 .LT. 0.0) THEN
GM1 = CMPLX ( 0.0,G1 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX(0.0,COTT1)
COT3 = CMPLX(0.0,COTT3)
ELSE
GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
END IF
G2=SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH/DH)*YH
ZE = NE/DE
ZZX1 = -1.*(ALPA*BETA)/(ALPA**2+BETA**2)
ZZX = ZZX1*(ZE-ZH)
X = ALPA*W/2.
Y = BETA*AL/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
```

```
      J3 = J31/J32
      J41 = COS(Y)/(Y)
      J42 = SIN(Y)/(Y**2)
      J4 = J41-J42
      JJ4 = J3*J4
      JX1 = CMPLX ( JJ4,0.0 )
      J11 = 2.*SIN(X)/(X)
      J12 = 3./(X**2)
      J13 = COS(X)
      J14 = 2.*SIN(X)/(X)
      J15 = (2.*(1.-COS(X)))/(X**2)
      J1 = J11+J12*(J13-J14+J15)
      J21 = PI*COS(Y)
      J22 = (PI/2)**2-Y**2
      J2 = J21/J22
      JJ1 = J1*J2
      JZ1 = CMPLX (JJ1,0.0 )
      F2 = ZZX*JX1*JZ1
      RETURN
      END

      COMPLEX FUNCTION F3(ALPA,BETA,FREQ)
      COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
      COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
      COMPLEX ZXZ,JZ1,JX1
      REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
      REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,M0,PI,ER,E0
      REAL X,Y,J11,J12,J13,J14,J15,J1,J21,J22,J2,JJ1
      REAL J41,J42,J4,J31,J32,J3,JJ4,ZXZ1
      PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12 )
      PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
      PARAMETER ( M0=PI*4.E-7, AL=0.01 )
      PARAMETER ( A=1.6E-03, W=0.711E-03 )
*     NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
*     NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

      YYE = -1./(2.E+9*PI*FREQ*E0)
      YE = CMPLX (0.0,YYE)
      YYH = 2.E+9*PI*M0*FREQ
      YH = CMPLX (0.0,YYH)
      AK1 = M0*E0*((2.E+9*PI*FREQ)**2)
      AK2 = M0*E0*ER*((2.E+9*PI*FREQ)**2)
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX(0.0,COTT1)
      COT3 = CMPLX(0.0,COTT3)
      ELSE
```

```
GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G3*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
ENDIF
G2=SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH / DH) * YH
ZE = NE / DE
ZXZ1 = -1.*(ALPA*BETA)/(ALPA**2 + BETA**2)
ZXZ = ZXZ1*(ZE-ZH)
X = ALPA*W/2.
Y = BETA*AL / 2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = (2.0*(1.0-COS(X))) / (X**2)
J1 = J11+J12*(J13-J14+J15)
J21 = PI*COS(Y)
J22 = (PI/2.)**2 - (Y**2)
J2 = J21/J22
JJ1 = J1*J2
JZ1 = CMPLX ( JJ1,0.0 )
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31 / J32
J41 = COS(Y)/(Y)
J42 = SIN(Y)/(Y**2)
J4 = J41-J42
JJ4 = J3*J4
JX1 = CMPLX ( JJ4,0.0 )
F3 = ZXZ*JZ1*JX1
RETURN
END
```

```
      COMPLEX FUNCTION F4(ALPA,BETA,FREQ)
      COMPLEX ZE,ZH,DE,NE,NH,COT1,COT2,COT3
      COMPLEX GM1,GM2,GM3,YE,YH,DH1,DH2,DH
      COMPLEX ZXX1,ZXX2,ZXX,JX1
      REAL G1,G2,G3,AK1,AK2,AKK1,AKK2,GMM1,GMM2,GMM3,COTT1,COTT2,COTT3
      REAL YYE,YYH,ALPA,BETA,FREQ,AL,H,T,D,W,A,MO,PI,ER,EO
      REAL X,Y,X1,J31,J32,J3,J41,J42,J4,JJ4
      PARAMETER ( PI=3.141592654, ER=2.2, EO=8.85E-12 )
      PARAMETER ( H=0.66E-03, T=0.255E-03, D=0.66E-03 )
      PARAMETER ( MO=PI*4.E-7, AL=0.01 )
      PARAMETER ( A=1.6E-03, W=0.711E-03 )
*     NOTE : PAREMETER W CAN BE 0.711, 1.0, OR 1.25 mm
*     NOTE : PAREMETER ER CAN BE 2.2 OR 9.8

      YYE = -1./(2.E+9*PI*FREQ*EO)
      YE = CMPLX (0.0,YYE)
      YYH = 2.E+9*PI*MO*FREQ
      YH = CMPLX (0.0,YYH)
      AK1 = MO*EO*((2.E+9*PI*FREQ)**2)
      AK2 = MO*EO*ER*((2.E+9*PI*FREQ)**2)
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX(0.0,COTT1)
      COT3 = CMPLX(0.0,COTT3)
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = 1./TANH(GMM1)
      COTT3 = 1./TANH(GMM3)
      COT1 = CMPLX ( COTT1,0.0 )
      COT3 = CMPLX ( COTT3,0.0 )
      ENDIF
      G2=SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX ( G2,0.0 )
      GMM2 = G2*T
      COTT2 = 1./TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
```

```
NE = ((GM2/ER)*COT3+GM3*COT2)*YE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1+DH2
ZH = (NH / DH) * YH
ZE = NE / DE
X1 = -1./(ALPA**2 + BETA**2)
ZXX1 = CMPLX (X1,0.0)
ZXX2 = (ALPA**2)*ZE + (BETA**2)*ZH
ZXX = ZXX1*ZXX2
X = ALPA*W/2.
Y = BETA*AL/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31 / J32
J41 = COS(Y)/(Y)
J42 = SIN(Y)/(Y**2)
J4 = J41-J42
JJ4 = J3*J4
JX1 = CMPLX ( JJ4,0.0 )
F4 = ZXX*JX1*JX1
RETURN
END
```

APPENDIX C. PROGRAMS FOR CALCULATION OF PROPAGATION CONSTANT

```
*       VARIABLE DEFINITION:
*       ====================
*       I    = REPRESENT THE REGION 1,2,3 IN RESONATOR
*       ALPA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (x-direction)
*       BETA = DISCRETE TRANSFORM VARIABLE FOR DOMINANT MODE (z-direction)
*       A    = A HALF WIDTH OF RESONATOR DEVICE = 1.6 mm
*       D    = HEIGHT OF RESONATOR BELOW SUBSTRATE = 0.66 mm
*       T    = THE THICKNESS OF SUBSTRATE = 0.255 mm
*       W    = THE WIDTH OF STRIP, CAN BE 0.711 mm, 1.0 mm, or 1.25 mm
*       H    = THE HEIGHT OF RESONATOR ABOVE SUBSTRATE = 0.66 mm
*       AL   = THE LENGTH OF STRIPLINE , ASSUME = 10 mm
*       AKI  = RADIAN FREQUENCY * SQRT OF (PERMITTIVITY*PERMEABILITY)
*       MO   = PERMEABILITY IN VACCUM = PI*4.E-7
*       MR   = PERMEABILITY IN A SUBSTRATE ASSUME = 1
*       EO   = PERMITTIVITY IN VACCUM = (1/(36*PI))*E-9 = 8.85E-12
*       ER   = PERMITIVITY IN A SUBSTRATE, CAN BE 2.2 OR 9.8
*       FREQ = FREQUENCY OPERATION IN GHZ
*       DE   = DENOMINATOR OF ZE
*       DE   = NUMERATOR OF ZE
*       DH1,DH2,DH = DENOMINATOR OF ZH
*       NH   = NUMERATOR OF ZH
*       ZE   = IMPEDANCE OF ELECTRIC FIELD, eqn (1g)
*       ZH   = IMPEDANCE OF MAGNETIC FIELD, eqn (1h)
*       ZZZ,ZZX,ZXZ,AND ZXX ARE THE GREEN'S IMPEDANCE FUNCTION (1c,d,e,f)
*       COT1 = CT1, EQN (1i)
*       COT2 = CT2, EQN (1j)
*       COT3 = CT3, EQN (1k)
*       J1   = EQN (8a)
*       J2   = EQN (8b)
*       JJ1  = Jz1   EQN (6a)
*       JJ4  = Jx1   EQN (6b)
*       JZ1  = BASIS FUNCTION OF CURRENT COMPONENT IN Z - DIRECTION
*       JX1  = BASIS FUNCTION OF CURRENT COMPONENT IN X - DIRECTION
*       K11,K12,K21,K22 = MATRIX ELEMENT
*       J1, J3 = ASSUME CURRENT DISTRIBUTION IN X - DIRECTION
*       J2, J4   = ASSUME CURRENT DISTRIBUTION IN Z - DIRECTION
*       ALL COMPUTATION IN STANDARD "MKS"
```

A. FIRST ORDER APPROXIMATION

```
*     THIS IS AN EXAMPLE PROGRAM TO COMPUTE CONSTANT PROPAGATION
*     IN FIRST ORDER APPROXIMATION WITH W = 0.711 AND ER = 2.2
      ==========================================================

*     MAIN PROGRAM :
*     ==============

*     FUNCTION : F1(ALPA,BETA,FREQ) = ZZZ * JZ1 ** 2
      COMPLEX F1,K11
      REAL B0,B00,MAGKK,ALPA,BETA,M0
      REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0
      INTEGER N,L
      PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
      PARAMETER ( W=0.711E-3 )
*     NOTE : PARAMETER W CAN BE 0.711 MM, 1.0 MM, 1.25 MM
*     NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

      FREQ = 12.9095783
      OMG = FREQ*PI*2.E+9
      B0 = (OMG/3.E08)*SQRT((ER+1)/2)

*     START SWEEP THE BETA VALUE FROM 84% TO 90%
      DO 20 B00 = 0.84,0.9,1.2E-3
      BETA = B0*B00
      K11 = (0.0,0.0)

*     START SUMMATION IN ALPA FROM N=-40 TO N=+40
      DO 10 L = 1, 81
      N = L - 41
      ALPA = (N - 0.5)*PI/A
      K11 = K11 + F1(ALPA,BETA,FREQ)
10    CONTINUE
      MAGKK = CABS((K11))
      WRITE(6,100) FREQ,BETA,MAGKK
100   FORMAT(' ',3X,F14.9,3X,F18.9,3X,F15.9)
20    CONTINUE
      STOP
      END

*     FUNCTION SUBPROGRAM F1
*     ======================

      COMPLEX FUNCTION F1(ALPA,BETA,FREQ)
      COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
      COMPLEX JZ1,ZZZ1,ZZZ2,ZZZ,YH
      REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YYE
      REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X1,X
      REAL J11,J12,J13,J14,J15,J1,M0,AK1,AK2,BETA
      PARAMETER ( PI=3.141592654, ER=2.2, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
```

```
      PARAMETER ( W=0.711E-3 )
*     NOTE : PARAMETER W CAN BE 0.711 MM, 1.0 MM, 1.25 MM
*     NOTE : PARAMETER ER CAN BE 2.2 OR 9.8

      OMG = FREQ*PI*2.E+9
      YYE = -1./(OMG*E0)
      YE = CMPLX (0.0,YYE)
      YYH = OMG*M0
      YH = CMPLX (0.0,YYH)
      AK1 = (OMG**2)*M0*E0
      AK2 = (OMG**2)*M0*E0*ER
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX ( 0.0,COTT1 )
      COT3 = CMPLX ( 0.0,COTT3 )
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G1*D
      COTT1 = 1./TANH(GMM1)
      COTT3 = 1./TANH(GMM3)
      COT1 = CMPLX ( COTT1,0.0 )
      COT3 = CMPLX ( COTT3,0.0 )
      END IF
      G2 = SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX ( G2,0.0 )
      GMM2 = G2*T
      COTT2 = 1./TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
      NE = (GM2*COT3/ER+GM3*COT2)*YE
      ZE = NE/DE
      NH = GM2*COT2 + GM3*COT3
      DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
      DH2 = GM2*GM3*COT2*COT3 + GM2**2
      DH = DH1 + DH2
      ZH = (NH/DH)*YH
      X1 = -1. / (ALPA**2 + BETA**2)
      ZZZ1 = CMPLX (X1,0.0)
```

```
ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
ZZZ = ZZZ1*ZZZ2
X = ALPA*W/2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = 2.*(1.0-COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
JZ1 = CMPLX ( J1,0.0 )
F1 = ZZZ*JZ1**2
RETURN
END
```

B. SECOND ORDER APPROXIMATION

```
*       THIS IS AN EXAMPLE PROGRAM TO COMPUTE CONSTANT PROPAGATION
*       IN SECOND ORDER APPROXIMATION WITH W = 0.711 AND ER = 2.2
*       =========================================================

*       MAIN PROGRAM :
*       ==============

*       F1(ALPA,BETA,FREQ) = ZZZ * JZ1 ** 2
*       F2(ALPA,BETA,FREQ) = ZZX * JZ1 * JX1
*       F3(ALPA,BETA,FREQ) = ZXZ * JZ1 * JX1
*       F4(ALPA,BETA,FREQ) = ZXX * JX1 ** 2

        COMPLEX F1,F2,F3,F4
        COMPLEX KK,K11,K22,K33,K44
        COMPLEX KKK1,KKK2
        REAL MAGKK,B00,B0,ALPA,BETA,M0
        REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0
        INTEGER N,L
        PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
        PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
        PARAMETER ( W=0.711 ,ER=2.2 )
*       NOTE : PARAMETER  W CAN BE 0.711 , 1.0 , OR 1.25
*       NOTE : PARAMETER  ER CAN BE 2.2 OR 9.8

        FREQ = 12.858241
        OMG = FREQ*PI*2.E+9
        B0 = (OMG/3.E08)*SQRT((ER+1)/2)

*       START SWEEP THE BETA VALUE FROM 88.2% TO 89.7%
        DO 20 B00 = 0.882,0.897,7.5E-6
        BETA = B0*B00
        K11 = (0.0,0.0)
        K22 = (0.0,0.0)
        K33 = (0.0,0.0)
        K44 = (0.0,0.0)

*       START SUMMATION IN ALPA FROM N=-40 TO N=+40
        DO 10 L = 1, 81
        N = L - 41
        ALPA = (N - 0.5)*PI/A
        K11 = K11 + F1(ALPA,BETA,FREQ)
        K22 = K22 + F2(ALPA,BETA,FREQ)
        K33 = K33 + F3(ALPA,BETA,FREQ)
        K44 = K44 + F4(ALPA,BETA,FREQ)
10      CONTINUE
        KKK1 = K11*K44
        KKK2 = K22*K33
        KK = KKK1-KKK2
        MAGKK = CABS((KK))
        WRITE(6,100) FREQ,BETA,MAGKK
100     FORMAT(' ',3X,F14.9,3X,F18.9,3X,F15.9)
20      CONTINUE
```

```
      STOP
      END

*     FUNCTIONS SUBPROGRAM F1, F2, F3, F4
*     ===================================

      COMPLEX FUNCTION F1(ALPA,BETA,FREQ)
      COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
      COMPLEX YH,JZ1,ZZZ1,ZZZ2,ZZZ
      REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YYE
      REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X1,X
      REAL J11,J12,J13,J14,J15,J1,M0,AK1,AK2,BETA
      PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
      PARAMETER ( W=0.711 ,ER=2.2 )
*     NOTE : PARAMETER  W CAN BE 0.711 , 1.0 , OR 1.25
*     NOTE : PARAMETER  ER CAN BE 2.2 OR 9.8

      OMG = FREQ*PI*2.E+9
      YYE = -1./(OMG*E0)
      YE = CMPLX (0.0,YYE)
      YYH = OMG*M0
      YH = CMPLX (0.0,YYH)
      AK1 = (OMG**2)*M0*E0
      AK2 = (OMG**2)*M0*E0*ER
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX ( 0.0,COTT1 )
      COT3 = CMPLX ( 0.0,COTT3 )
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G1*D
      COTT1 = 1./TANH(GMM1)
      COTT3 = 1./TANH(GMM3)
      COT1 = CMPLX ( COTT1,0.0 )
      COT3 = CMPLX ( COTT3,0.0 )
      END IF
      G2 = SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX ( G2,0.0 )
```

```
      GMM2 = G2*T
      COTT2 = 1./TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
      NE = (GM2*COT3/ER+GM3*COT2)*YE
      ZE = NE/DE
      NH = GM2*COT2 + GM3*COT3
      DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
      DH2 = GM2*GM3*COT2*COT3 + GM2**2
      DH = DH1 + DH2
      ZH = (NH/DH)*YH
      X1 = -1. / (ALPA**2 + BETA**2)
      ZZZ1 = CMPLX (X1,0.0)
      ZZZ2 = (BETA**2)*ZE + (ALPA**2)*ZH
      ZZZ = ZZZ1*ZZZ2
      X = ALPA*W/2.
      J11 = 2.*SIN(X)/(X)
      J12 = 3./(X**2)
      J13 = COS(X)
      J14 = J11
      J15 = 2.*(1.0-COS(X))/(X**2)
      J1 = J11+J12*(J13-J14+J15)
      JZ1 = CMPLX ( J1,0.0 )
      F1 = ZZZ*JZ1**2
      RETURN
      END

      COMPLEX FUNCTION F2(ALPA,BETA,FREQ)
      COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
      COMPLEX YH,JZ1,JX1,ZZX
      REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YYE
      REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X,ZZX1,BETA
      REAL J31,J32,J3,J11,J12,J13,J14,J15,J1,M0,AK1,AK2
      PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
      PARAMETER ( W=0.711 ,ER=2.2 )
*     NOTE : PARAMETER  W CAN BE 0.711 , 1.0 , OR 1.25
*     NOTE : PARAMETER  ER CAN BE 2.2 OR 9.8

      OMG = FREQ*PI*2.E+9
      YYE = -1./(OMG*E0)
      YE = CMPLX (0.0,YYE)
      YYH = OMG*M0
      YH = CMPLX (0.0,YYH)
      AK1 = (OMG**2)*M0*E0
      AK2 = (OMG**2)*M0*E0*ER
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
```

```
COTT1 = -1./TAN(GMM1)
COTT3 = -1./TAN(GMM3)
COT1 = CMPLX ( 0.0,COTT1 )
COT3 = CMPLX ( 0.0,COTT3 )
ELSE
GM1 = CMPLX ( G1,0.0 )
GM3 = GM1
GMM1 = G1*H
GMM3 = G1*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YE
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
ZZX1 = -1.*(ALPA*BETA)/(ALPA**2+BETA**2)
ZZX = ZZX1*(ZE-ZH)
X = ALPA*W/2.
J11 = 2.*SIN(X)/(X)
J12 = 3./(X**2)
J13 = COS(X)
J14 = J11
J15 = 2.*(1.0-COS(X))/(X**2)
J1 = J11+J12*(J13-J14+J15)
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31/J32
JZ1 = CMPLX ( J1,0.0 )
JX1 = CMPLX(J3,0.0)
F2 = ZZX*JZ1*JX1
RETURN
END

COMPLEX FUNCTION F3(ALPA,BETA,FREQ)
COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
COMPLEX YH,JZ1,JX1,ZXZ
REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YYE
```

```
      REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,X,BETA
      REAL J11,J12,J13,J14,J15,J1,J31,J32,J3,M0,AK1,AK2
      PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
      PARAMETER ( W=0.711 ,ER=2.2 )
*     NOTE : PARAMETER  W CAN BE 0.711 , 1.0 , OR 1.25
*     NOTE : PARAMETER  ER CAN BE 2.2 OR 9.8

      OMG = FREQ*PI*2.E+9
      YYE = -1./(OMG*E0)
      YE = CMPLX (0.0,YYE)
      YYH = OMG*M0
      YH = CMPLX (0.0,YYH)
      AK1 = (OMG**2)*M0*E0
      AK2 = (OMG**2)*M0*E0*ER
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX ( 0.0,COTT1 )
      COT3 = CMPLX ( 0.0,COTT3 )
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G1*D
      COTT1 = 1./TANH(GMM1)
      COTT3 = 1./TANH(GMM3)
      COT1 = CMPLX ( COTT1,0.0 )
      COT3 = CMPLX ( COTT3,0.0 )
      END IF
      G2 = SQRT(ABS(AKK2))
      IF (AKK2 .LT. 0.0) THEN
      GM2 = CMPLX ( 0.0,G2 )
      GMM2 = G2*T
      COTT2 = -1./TAN(GMM2)
      COT2 = CMPLX ( 0.0,COTT2 )
      ELSE
      GM2 = CMPLX ( G2,0.0 )
      GMM2 = G2*T
      COTT2 = 1./TANH(GMM2)
      COT2 = CMPLX ( COTT2,0.0 )
      END IF
      DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
      NE = (GM2*COT3/ER+GM3*COT2)*YE
      ZE = NE/DE
      NH = GM2*COT2 + GM3*COT3
      DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
      DH2 = GM2*GM3*COT2*COT3 + GM2**2
```

```
      DH = DH1 + DH2
      ZH = (NH/DH)*YH
      ZXZ = -1.*(ALPA*BETA)/(ALPA**2+BETA**2)
      ZXZ = ZXZ*(ZE-ZH)
      X = ALPA*W/2.
      J31 = 2.*PI*SIN(X)
      J32 = X**2 - PI**2
      J3 = J31/J32
      JX1 = CMPLX(J3,0.0)
      J11 = 2.*SIN(X)/(X)
      J12 = 3./(X**2)
      J13 = COS(X)
      J14 = J11
      J15 = 2.*(1.0-COS(X))/(X**2)
      J1 = J11+J12*(J13-J14+J15)
      JZ1 = CMPLX ( J1,0.0 )
      F3 = ZXZ*JZ1*JX1
      RETURN
      END

      COMPLEX FUNCTION F4(ALPA,BETA,FREQ)
      COMPLEX GM1,GM2,GM3,COT1,COT2,COT3,DE,ZE,DH,ZH,NE,NH,DH1,DH2,YE
      COMPLEX YH,JX1,ZXX1,ZXX2,ZXX
      REAL A,D,T,H,PI,ER,W,FREQ,OMG,E0,G1,G2,G3,COTT1,COTT2,COTT3,YYE
      REAL YYH,GMM1,GMM2,GMM3,ALPA,AKK1,AKK2,AK1,AK2
      REAL J31,J32,J3,M0,X,X1,BETA
      PARAMETER ( PI=3.141592654, E0=8.85E-12, H=0.66E-03 )
      PARAMETER ( T=0.255E-03, D=0.66E-03, M0=PI*4.E-7, A=0.16E-02 )
      PARAMETER ( W=0.711 ,ER=2.2 )
*     NOTE : PARAMETER  W CAN BE 0.711 , 1.0 , OR 1.25
*     NOTE : PARAMETER  ER CAN BE 2.2 OR 9.8

      OMG = FREQ*PI*2.E+9
      YYE = -1./(OMG*E0)
      YE = CMPLX (0.0,YYE)
      YYH = OMG*M0
      YH = CMPLX (0.0,YYH)
      AK1 = (OMG**2)*M0*E0
      AK2 = (OMG**2)*M0*E0*ER
      AKK1 = ALPA**2 + BETA**2 - AK1
      AKK2 = ALPA**2 + BETA**2 - AK2
      G1 = SQRT(ABS(AKK1))
      G3 = G1
      IF (AKK1 .LT. 0.0) THEN
      GM1 = CMPLX ( 0.0,G1 )
      GM3 = GM1
      GMM1 = G1*H
      GMM3 = G3*D
      COTT1 = -1./TAN(GMM1)
      COTT3 = -1./TAN(GMM3)
      COT1 = CMPLX ( 0.0,COTT1 )
      COT3 = CMPLX ( 0.0,COTT3 )
      ELSE
      GM1 = CMPLX ( G1,0.0 )
      GM3 = GM1
      GMM1 = G1*H
```

```
GMM3 = G1*D
COTT1 = 1./TANH(GMM1)
COTT3 = 1./TANH(GMM3)
COT1 = CMPLX ( COTT1,0.0 )
COT3 = CMPLX ( COTT3,0.0 )
END IF
G2 = SQRT(ABS(AKK2))
IF (AKK2 .LT. 0.0) THEN
GM2 = CMPLX ( 0.0,G2 )
GMM2 = G2*T
COTT2 = -1./TAN(GMM2)
COT2 = CMPLX ( 0.0,COTT2 )
ELSE
GM2 = CMPLX ( G2,0.0 )
GMM2 = G2*T
COTT2 = 1./TANH(GMM2)
COT2 = CMPLX ( COTT2,0.0 )
END IF
DE = COT2*COT3+COT1*COT3*(GM2/GM1)/ER+COT1*COT2+(GM3/GM2)*ER
NE = (GM2*COT3/ER+GM3*COT2)*YE
ZE = NE/DE
NH = GM2*COT2 + GM3*COT3
DH1 = GM1*GM2*COT1*COT2 + GM1*GM3*COT1*COT3
DH2 = GM2*GM3*COT2*COT3 + GM2**2
DH = DH1 + DH2
ZH = (NH/DH)*YH
X1 = -1./(ALPA**2 + BETA**2)
ZXX1 = CMPLX(X1,0.0)
ZXX2 = (ALPA**2)*ZE + (BETA**2)*ZH
ZXX = ZXX1*ZXX2
X = ALPA*W/2.
J31 = 2.*PI*SIN(X)
J32 = X**2 - PI**2
J3 = J31/J32
JX1 = CMPLX(J3,0.0)
F4 = ZXX*JX1**2
RETURN
END
```

APPENDIX D. PROGRAMS FOR CALCULATION OF FRINGING CAPACITANCE

A. TRANSMISSION LINE CIRCUIT MODEL

```
*     THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE FRINGING CAPACITANCE
*     IN SECOND ORDER APPROXIMATION WITH W = 0.711 MM AND ER = 2.2
*     ==============================================================
      COMPLEX ZIN
      REAL MAGZIN,OMG,Z0,BETA,AL,C1,C,FREQ,T,ZZ,PI,ZZIN
      PARAMETER ( PI=3.141592654, Z0=94.088, AL=0.005 )
      PARAMETER ( BETA = 301.921744 )
*     NOTE : FILL IN PARAMETERS BETA AND Z0 FROM TABLE 1 OR TABLE 2.
      DO 10 C1 = 0.8058,0.8062,1.E-6
      C = C1*1.E-14
      DO 15 FREQ = 12.858241,12.858241
      OMG = 2*PI*FREQ*1.E9
      T = TAN(BETA*AL)
      ZZ = OMG*C*Z0
      ZZIN = -1.*(1-ZZ*T)*Z0/(ZZ+T)
      ZIN = CMPLX(0.0,ZZIN)
      MAGZIN = CABS((ZIN))
      WRITE(6,100) FREQ,C,MAGZIN
100   FORMAT(' ',2X,F16.12,2X,F25.23,2X,F14.12)
15    CONTINUE
10    CONTINUE
      END
```

B. END EFFECT EXTENSION MODEL

```
*     THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE FRINGING CAPACITANCE
*     IN SECOND ORDER APPROXIMATION WITH W = 0.711 MM AND ER = 2.2
*     ==============================================================
      REAL F0,BETA,LL,L,PI,DELTA,C0,OMG,Z0,CP
      PARAMETER ( Z0=94.088, L=0.01, PI=3.141592654 )
      PARAMETER ( F0=12.858241, BETA=301.921744 )
*     NOTE : PARAMETERS F0, BETA, AND Z0 CAN VARIES DEPEND ON THE CHOICE
*     OF APPROXIMATION
      OMG   = 2.*PI*F0*1.E9
      LL    = PI/BETA
      DELTA = (LL-L)/2.
      C0 = BETA/(OMG*Z0)
      CP = C0*DELTA
      WRITE(6,100) F0,BETA,CP
100   FORMAT(' ',3X,F10.7,5X,F9.5,3X,F30.29)
      END
```

APPENDIX E. PROGRAM FOR CALCULATION OF CHARACTERISTIC IMPEDANCE OF SUSPENDED SUBSTRATE LINE

```
*       THIS IS AN EXAMPLE PROGRAM TO COMPUTE THE DYNAMIC
*       CHARACTERISTIC IMPEDANCE OF SUSPENDED SUBSTRATE LINE
*       IN SECOND ORDER APPROXIMATION WITH W=1.25 & ER=9.8
*       ==================================================


        REAL EEFF,OMG,FREQ,BETA,W,G,ER,T,A,Z0,Z11,Z1,V,R
        PARAMETER ( W=1.25E-3, G=1.575E-3, ER=9.8, T=0.255E-3 )
        PARAMETER ( A=3.2E-3 , BETA = 293.61813 , PI=3.141592654)
*       NOTE : PARAMETER A IN FIGURE 1 IS 2A = 3.2E-3 METER

        FREQ = 9.78036
        V = -1.7866-0.2035*(T/G)+0.4750*(A/G)
        R = 1.0835+0.1007*(T/G)-0.09457*(A/G)
        Z11 = 6*G/W + (1+4*((G/W)**2))**0.5
        Z1 = 60*(V+R*LOG(Z11))
        OMG = FREQ*PI*2.E+9
        EEFF = (BETA*3.E+8/OMG)**2
        Z0 = Z1/(EEFF**0.5)
        WRITE(6,25) Z0
25      FORMAT(' ',5X,F9.3)
        STOP
        END
```

APPENDIX F. SOME MATHEMATICAL CALCULATION

These equations are used in computer programs

1. ***The equations (1p,q,m) on page 7***

$$Y_1 = j\omega\varepsilon_o \qquad Z_1 = j\omega\mu_o \qquad K_1^2 = \omega^2\mu_o\varepsilon_o \tag{1}$$

$$Y_2 = j\omega\varepsilon_o\varepsilon_r \qquad Z_2 = j\omega\mu_o\mu_r \qquad K_2^2 = \omega^2\mu_o\mu_r\varepsilon_o\varepsilon_r \tag{2}$$

$$Y_3 = j\omega\varepsilon_o \qquad Z_3 = j\omega\mu_o \qquad K_3^2 = \omega^2\mu_o\varepsilon_o \tag{3}$$

Note that $Y_1 = Y_3,\ Z_1 = Z_3,\ K_1^2 = K_3^2$

2. ***The equations (1n,o) on page 7***

$$\gamma_{y1} = \frac{\gamma_1}{Y_1} = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega\varepsilon_o} \qquad \gamma_{z1} = \frac{\gamma_1}{Z_1} = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega\mu_o} \tag{4}$$

$$\gamma_{y2} = \frac{\gamma_2}{Y_2} = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\varepsilon_o\varepsilon_r} \qquad \gamma_{z2} = \frac{\gamma_2}{Z_2} = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\mu_o\mu_r} \tag{5}$$

$$\gamma_{y3} = \frac{\gamma_3}{Y_3} = \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\varepsilon_o} \qquad \gamma_{z3} = \frac{\gamma_3}{Z_3} = \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\mu_o} \tag{6}$$

3. ***The equation (1g) on page 6***

$$ZE = \frac{NE}{DE} \tag{7}$$

Where :

$$NE = \gamma_{y2}Ct_3 + \gamma_{y3}Ct_2 \tag{8}$$

$$NE = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\varepsilon_o\varepsilon_r} \times Ct_3 + \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\varepsilon_o} \times Ct_2 \tag{9}$$

$$NE = \frac{1}{j\omega\varepsilon_o}\left[\,\frac{Ct_3}{\varepsilon_r}\sqrt{\alpha^2 + \beta^2 - K_2^2} + Ct_2\sqrt{\alpha^2 + \beta^2 - K_3^2}\,\right]$$

$$NE = \frac{1}{j\omega\varepsilon_o}\left[\frac{Ct_3}{\varepsilon_r} \times \gamma_2 + Ct_2 \times \gamma_3\right] \tag{10}$$

Defined $\frac{1}{j\omega\varepsilon_o}$ = YY'., then

$$YYE = \frac{1}{j\omega\varepsilon_o} = j\frac{-1}{\omega\varepsilon_o} = j\{\frac{-1}{2\pi f\varepsilon_o}\}$$

$$YE = Complex(0.0,\ YYE) \tag{11}$$

$$DE = DE1 + DE2$$

$$DE1 = Ct_2Ct_3 + Ct_1Ct_3\left(\frac{\sqrt{\alpha^2+\beta^2-K_2^2}}{j\omega\varepsilon_o\varepsilon_r}\right)/\left(\frac{\sqrt{\alpha^2+\beta^2-K_1^2}}{j\omega\varepsilon_o}\right) \tag{12}$$

$$DE2 = Ct_1Ct_2\frac{\gamma_{y3}}{\gamma_{y1}} + \left(\frac{\sqrt{\alpha^2+\beta^2-K_3^2}}{j\omega\varepsilon_o}\right)/\left(\frac{\sqrt{\alpha^2+\beta^2-K_2^2}}{j\omega\varepsilon_o\varepsilon_r}\right) \tag{13}$$

Since $\gamma_{y3} = \gamma_{y1}$, simplicity (12) and (13)

$$DE1 = Ct_2Ct_3 + Ct_1Ct_3\frac{\sqrt{\alpha^2+\beta^2-K_2^2}}{\sqrt{\alpha^2+\beta^2-K_1^2}} \times \frac{1}{\varepsilon_r}$$

$$DE1 = Ct_2Ct_3 + Ct_1Ct_3\frac{\gamma_2}{\gamma_1} \times \frac{1}{\varepsilon_r} \tag{14}$$

$$DE2 = Ct_1Ct_2\frac{\sqrt{\alpha^2+\beta^2-K_3^2}}{\sqrt{\alpha^2+\beta^2-K_2^2}} \times \varepsilon_r$$

$$DE2 = Ct_1Ct_2\frac{\gamma_3}{\gamma_2} \times \varepsilon_r \tag{15}$$

$$DE = Ct_2Ct_3 + Ct_1Ct_3 \times \frac{\gamma_2}{\gamma_1} \times \frac{1}{\varepsilon_r} + Ct_1Ct_2 \times \frac{\gamma_3}{\gamma_2} \times \varepsilon_r \tag{16}$$

In the programs use a symbols:
COT1, COT2, COT3 for Ct_1, Ct_2, Ct_3, and
GM1, GM2, GM3 for $\gamma_1, \gamma_2, \gamma_3$.

4. The equation (1i) on page 6

$$Ct_1 = Coth\ \gamma_1 h = Coth\ \{\sqrt{\alpha^2+\beta^2-K_1^2} \times h\} \tag{17}$$

IF $(\alpha^2 + \beta^2 - K_1^2) < 0.0$, then γ_1 is complex. We defined γ_1 as G_1, so

$$Ct_1 = Coth\ jG_1h = \frac{1}{Tanh\ jG_1h} \tag{18}$$

$$Ct_1 = \frac{1}{jTan\ G_1h} = \frac{-j}{Tan\ G_1h} \tag{19}$$

$$Ct_1 = Complex(0.0, \frac{-1}{Tan\ G_1h}) \tag{20}$$

ELSEIF

$$Ct_1 = Coth\ G_1h = \frac{1}{\tanh G_1h} \tag{21}$$

$$Ct_1 = Complex(\frac{1}{Tanh\ G_1h}, 0.0) \tag{22}$$

The same process for Ct_2 and Ct_3 in equation (1j) and (1k).

5. **The equation (1h) on page 6**

$$ZH = \frac{NH}{DH} \tag{23}$$

Where :

$$NH = \gamma_{z2}Ct_2 + \gamma_{z3}Ct_3 \tag{24}$$

$$NH = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\mu_o\mu_r} \times Ct_2 + \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\mu_o} \times Ct_3 \tag{25}$$

$$NH = \frac{1}{j\omega\mu_o}[\frac{Ct_2}{\mu_r}\sqrt{\alpha^2 + \beta^2 - K_2^2} + Ct_3\sqrt{\alpha^2 + \beta^2 - K_3^2}]$$

We assumed that the value of $\boldsymbol{\mu_r}$ = 1.0, then we have

$$NH = \frac{1}{j\omega\mu_o}[Ct_2 \times \gamma_2 + Ct_3 \times \gamma_3] \tag{26}$$

Write NH in a symbolic form,

$$NH = \frac{1}{j\omega\mu_o} \times [\&\&\&] \tag{27}$$

$$DH = \gamma_{z1}\gamma_{z2}Ct_1Ct_2 + \gamma_{z1}\gamma_{z3}Ct_1Ct_3 + \gamma_{z2}\gamma_{z3}Ct_2Ct_3 + \gamma_{z2}^2 \tag{28}$$

$$DH1 = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega\mu_o} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\mu_o\mu_r} \times Ct_1Ct_2 \tag{29}$$

$$DH2 = \frac{\sqrt{\alpha^2 + \beta^2 - K_1^2}}{j\omega\mu_o} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\mu_o} \times Ct_1 Ct_3 \tag{30}$$

$$DH3 = \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\mu_o\mu_r} \times \frac{\sqrt{\alpha^2 + \beta^2 - K_3^2}}{j\omega\mu_o} \times Ct_2 Ct_3 \tag{31}$$

$$DH4 = [\ \frac{\sqrt{\alpha^2 + \beta^2 - K_2^2}}{j\omega\mu_o\mu_r}\ ]^2 \tag{32}$$

Simplify (29 to 32) :

$$DH1 = \frac{1}{(j\omega\mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times Ct_1 Ct_2 \tag{33}$$

$$DH2 = \frac{1}{(j\omega\mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} \times Ct_1 Ct_3 \tag{34}$$

$$DH3 = \frac{1}{(j\omega\mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} \times Ct_2 Ct_3 \tag{35}$$

$$DH4 = \frac{1}{(j\omega\mu_o)^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} \tag{36}$$

$$DH = \frac{1}{(j\omega\mu_o)^2} \times (DH1 + DH2 + DH3 + DH4) \tag{37}$$

Write DH in a symbolic form,

$$DH = \frac{1}{(j\omega\mu_o)^2} \times [SSS] \tag{38}$$

We defined in the program that $K_1^2 = K_3^2 = AK1$ and $K_2^2 = AK2$

$$\sqrt{\alpha^2 + \beta^2 - K_1^2} \times \sqrt{\alpha^2 + \beta^2 - K_3^2} = AKK1$$

$$\alpha^2 + \beta^2 -- AK_1 = AKK1 \tag{39}$$

and

$$\sqrt{\alpha^2 + \beta^2 - K_2^2} \times \sqrt{\alpha^2 + \beta^2 - K_2^2} = AKK2$$

$$\alpha^2 + \beta^2 - AK_2 = AKK2 \tag{40}$$

From these equations we have the value of ZH as the following:

$$ZH = \frac{\frac{1}{j\omega\mu_o} \times [\&\&\&]}{(\frac{1}{j\omega\mu_o})^2 \times [SSS]} = j\omega\mu_o \times \frac{[\&\&\&]}{[SSS]} \qquad (41)$$

Defined $j\omega\mu_o$ = YYH

$$YH = Complex(0.0,\ YYH) \qquad (43)$$

So

$$ZH = YH \times \frac{[\&\&\&]}{[SSS]}$$

LIST OF REFERENCES

1. Tatsuo Itoh, "Analysis of Microstrip Resonators", *IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-22*, November 1974.

2. Tatsuo Itoh, *Numerical Techniques for Microwave Passive Structures*, pp. 334-342, John Wiley & Sons, 1989.

3. Tatsuo Itoh and R.Mittra, "A Technique for Computing Dispersion Characteristics of Shielded Microstrip Lines", *IEEE Transaction on Microwave Theory and Techniques*, October 1974.

4. E. Yamashita and R. Mittra, "Variational Method for the Analysis of Microstrip Lines", *IEEE Transaction on Microwave Theory and Techniques, Vol. MTT-16*, April 1968.

5. E. Yamashita, "Variational Method for the Analysis of Microstrip-Like Transmission Lines", *IEEE Transaction on Microwave Theory and Techniques, Vol. MTT-16 No.8*, August 1968.

6. P. Silvester, "Equivalent Capacitances of Microstrip Open Circuits", *IEEE Transaction on Microwave Theory and Techniques, Vol. MTT-20*, 1972.

7. Harry A. Atwater, *Introduction to Microwave Theory*, Robert E. Krieger Publishing Co., 1981.

8. Rome Air Development Center, "Computation of Lumped Microstrip Capacities by Matrix Methods - Rectangular Section and End Effect", *IEEE Transaction on Microwave Theory and Techniques*, May 1971.

9. Yong-hui Shu, Xiao-xia Qi, Yun-yi Wang, "Analysis Equations for Shielded Suspended Substrate Microstrip Line and Broadside-Coupled Stripline", *IEEE MTT Digest*, pp. 693-695, 1987.

10. Fred E. Gardiol, "Careful MIC Design Prevents Waveguide Modes", *Microwaves*, pp. 188-191, May 1977.

11. David Rubin and Alfred R. Hislop, "Design Techniques for Suspended Substrate and Microstrip", *Microwave Journal*, October 1980.

12. Robert Grover Brown, Robert A. Sharpe, William E. Post, *Line, Waves, and Antennas*, John Wiley & Sons, 1973.

13. Peter A. Rizzi, *Microwave Engineering Passive Circuits*, Prentice-Hall, Inc. 1988.

14. Raj Mittra and Tatsuo Itoh, "A New Technique for the Analysis of the dispersion Characteristics of Microstrip Lines", *IEEE Transaction on Microwave Theory and Techniques, Vol. MTT-19, No. 1*, pp. 47-56, January 1971.

15. E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines", *IEEE Transaction on Microwave Theory and Techniques, Vol. MTT-19*, pp. 30-39, January 1971.

16. T. Itoh, R. Mittra, and R.D. Ward, " A method for computing edge capacitance of finite and semi-infinite microstrip lines", *IEEE Transaction on Microwave Theory and Techniques, (Short paper), Vol. MTT-20*, pp. 847-849, December 1972.

17. Yong-hui Shu, Xiao-xia Qi, Yun-yi Wang, "Closed form equation for Shielded Suspended Substrate Line", *Appendix of [Ref. 9]*

18. Tomoki Uwano, "Accurate Characterization of Microstrip Resonator Open End with New Current Expression in Spectral-Domain Approach", *IEEE Transaction on Microwave Theory and Techniques, Vol. 37, No. 3*, pp. 630-633, March 1989.

19. A.K. Kotronis, "Millimeter Wave Filter Design with Suspended Substrate Line", *M.S. Thesis*, 1989, Naval Postgraduate School, Monterey, CA.

20. Clifford M. Krowne, "Relationships for Green,s Function Spectral Dyadics Involving Anisotropic Imperfect Conductors Imbedded in Layered Anisotropic Media", *IEEE Transaction on Antennas and Propagation, Vol. 37, No. 9*, pp. 1207 1210, September 1989.

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